

# MULTI-FLUID HYDRODYNAMICS IN CHARGE DENSITY WAVES WITH COLLECTIVE, ELECTRONIC, AND SOLITONIC DENSITIES AND CURRENTS

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**Introduction: CDWs and their intrinsic defects.** Charge density waves (CDW) are spontaneous periodic superstructures  $\propto A \exp(\mathbf{q}_0 \cdot \mathbf{r} + \varphi)$  which are ubiquitous in quasi-1D electronic systems, see the latest review [1]. The translational degeneracy of the incommensurate CDW ground state allows for formation of topological defects: local ones, like phase and amplitude solitons (see [2] for the literature review) and extended ones, like planes of domain walls as solitonic lattices [3, 4], lines or loops of dislocations as phase vortices [5–7]. Experimentally, their presence was identified by various methods; just mention the direct visualization of solitons by the STM [8, 9] and indications on dislocations from the coherent X-ray microdiffraction [10] and from reconstruction in mesa-junctions [11].

The motion of dislocations is allowed only as a matter conserving glide along the chains, in the direction of the Burgers vector  $\mathbf{b} = 2\pi(1, 0, 0)$ . The transversal motion, the non conserving climb, is prohibited whatever is the driving force coming from the local stress — in a strong difference with respect to conventional vortices. In CDWs, the climb may be allowed by the condensation or liberation of normal carriers providing the conversion between normal and collective currents. Otherwise, even lacking the topological protection, the pairs or rings of dislocations do not annihilate as in

the conventional  $xy$  model and the complex-field theory but are stabilized by the matter (here the number of condensed electrons) conservation law. The minimal dislocation loops are the charge  $\pm 2e$  objects in a form of  $\pm 2\pi$  solitons: in a discrete view of the quasi-1D system, here the CDW at the defected chain gains or loses one period with respect to surrounding chains. These phase solitons were assumed to be seen experimentally as lowest activation charge carriers (see [12] and Refs. therein).

Describing the coexistence of electrons and defects in static equilibrium, under strains and in the current carrying state requires for a general nonlinear hydrodynamics for two fields — the phase and the electric potential and for two fluids of electrons and defects. This article suggests a contribution to this request.

**Results and conclusions.** At presence of topological defects the local deformations and velocities  $\omega_j$ ,  $j = x, y, z, t$ , cannot be derivatives of the same phase  $\varphi$ . Our key observation was an existence of a uniquely defined and allowed for averaging phase  $\chi$  which derivatives are given as

$$\begin{aligned} \partial_t \chi &= \langle \omega_t \rangle + 2\pi j_d, & \partial_x \chi &= \langle \omega_x \rangle - 2\pi n_d, \\ \partial_y \chi &= \langle \omega_y \rangle, & \partial_z \chi &= \langle \omega_z \rangle, \end{aligned} \quad (1)$$

where  $n_d$  is the concentration of defects — the mean area of loops per volume (taking vorticity signs into account).

We employed the local energy functional appropriate to CDWs as described in [13]. Equations for the

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average phase  $\chi$  and the potential  $\Phi$  have been derived as

$$(\hat{\Delta} - \gamma \partial_t) \chi = F_{pin} + E - 2\gamma j_d - \partial_x(n_n + 2n_d), \quad (2)$$

$$r_0^2 \Delta \Phi + \partial_x \chi + 2n_d + n_n = 0, \quad E = -\partial_x \Phi, \quad (3)$$

where  $\hat{\Delta} = \partial_x^2 + \Delta_\perp$ ,  $\Delta_\perp = \alpha_y \partial_y^2 + \alpha_z \partial_z^2$ ,  $\alpha_{y,z}$  are anisotropy coefficients coming from the interchain coupling of CDWs and  $r_0$  is the Tomas-Fermi radius of the parent metal; the concentration  $n_n$  of normal carriers will be neglected from now on. Eq. (2) shows that the phase  $\chi$  is driven, in addition to standard forces  $F_{pin} + E$ , also by the current of defects and by the longitudinal gradient of the total number of particles. Eq. (2) shows that the density and the current of defects contribute in the frame of the average phase  $\chi$ , contrarily to being obscure in the frame of local independent distortions  $\omega_i$ .

Equations (2), (3) must be complemented by the laws governing the distribution of defects. It was important to realize that the force driving the glide of defect comes only from share strains  $F_d = 2\hat{\Delta}\chi$ . In the diffusion approximation, we get

$$(\hat{\Delta} - \gamma \partial_t) \chi + 2\gamma b_d n_{d,tot} \hat{\Delta}_\perp \chi + 2(\gamma D_d + 1) \partial_x n_d = F_{pin} + E, \quad (4)$$

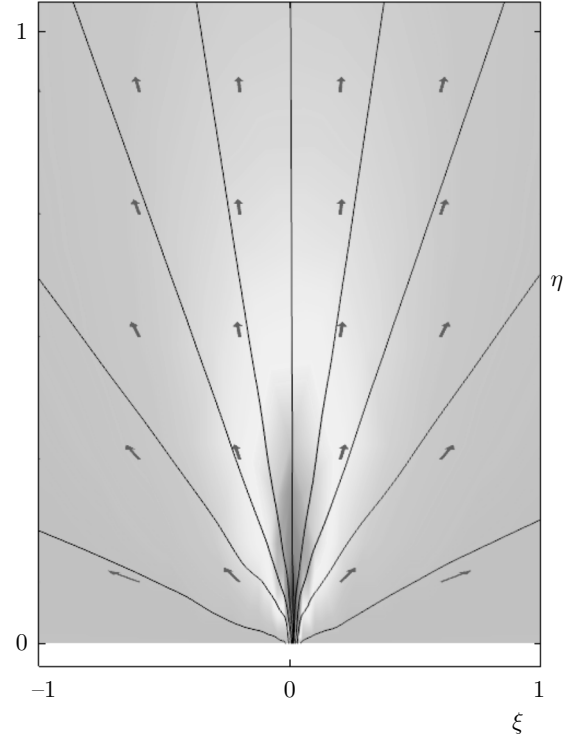
where  $b_d$  and  $D_d = b_d T$  are the mobility and the diffusion coefficient of defects. The allowance for defects' motion contributes additively to the transverse rigidity  $\propto \hat{\Delta}_\perp \chi$  of the phase and to the effective field from the gradient of defects' concentration.

The derived equations can be generalized to take into account isolated dislocation lines embedded to the averaged ensemble of dislocation loops. For a static dislocation directed in  $z$  and centered at  $(0,0)$ , the equations can yield

$$[r_0^2 \alpha \partial_y^4 - \partial_x^2] \chi - 2\partial_x n_d = \partial_x \delta(x) \text{Sgn}(y). \quad (5)$$

This equation was derived from Eqs. (2), (3) in the electroneutrality approximation realizing that  $r_0$  is the smallest length scale. Here, the conventional Laplacian form for an anisotropic elastic media acquired a nonanalytic dominating contribution of an anomalous elasticity [6] coming from long range Coulomb forces. The defects concentration is regulated by their dependence on the chemical potential  $\zeta_d$  which obeys the equilibrium condition  $j_d = 0$  as

$$n_d = n_\infty \text{sh} \frac{\zeta}{T}, \quad j_d \propto \alpha_y \partial_y^2 \chi - \partial_x \zeta_d = 0. \quad (6)$$



(Color online) Distributions around a dislocation centered at  $(0,0)$ .  $\xi, \eta$  are dimensionless rescaled coordinates  $x, y$ . Vectors and streamlines characterize the phase  $\chi$ . The color indicates the chemical potential  $\zeta = ZT$ .  $Z$  changes from  $Z \approx 0$  at large distances (green color) to a maximal value  $Z \approx 2.5$  near the origin (red color) and then drops to zero (blue color)

Results of a numerical solution of the above equations are illustrated in Figure. The conventional rotation of the phase following the coordinate angle at large distances becomes near the core a nearly vertical drop indicating the high  $x$ -gradient in accordance with the rapidly growing  $Z = \zeta/T$ . The enhancement of  $Z$  up to  $Z \approx 2.5$  corresponding to increasing of solitons' concentration near the dislocation by a factor  $n(\text{core})/n_\infty \approx 6$ .

In summary, we have derived general equations for the multi-fluid hydrodynamics of plastic flows with collective, electronic, and solitonic densities and currents. As an application, we presented distributions of fields around an isolated dislocation line in the regime of nonlinear screening by the gas of phase solitons.

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