ЖЭТФ, 2022, том 162, вып. 3 (9), стр. 373–376

HOLOGRAPHIC DESCRIPTION OF THE DISSIPATIVE MODEL OF UNIVERSE WITH CURVATURE

I. Brevik [a*], ***A. V. Timoshkin*** [b,c**]

[a] *Department of Energy and Process Engineering, Norwegian University of Science and Technology N-7491 Trondheim, Norway*

[b] *Institute of Scientific Research and Development, Tomsk State Pedagogical University (TSPU) 634061, Tomsk, Russia*

[c] *International Centre of Gravity and Cosmos, Tomsk State University of Control Systems and Radioelectronics (TUSUR) 634050, Tomsk, Russia*

Received April 21, 2022,
revised version April 29, 2022
Accepted May 4, 2022

DOI: 10.31857/S0044451022090097
EDN: EKQOFH

A description of the accelerated expansion of the Universe [1, 2] is possible with the help of dark energy or through modified gravity [3, 4]. Dark energy can be represented within the framework of the cosmological model using a dark fluid with an unusual equation of a state [5, 6].

The most realistic cosmological picture of the evolution of the Universe is obtained taking into account the properties of the viscosity of the dark fluid. This is due to the fact that in the era of dark energy the occurrence of cosmological singularities with the final time of formation is possible. The influence from (bulk) viscosity affects the behavior of the universe near cosmological singularities [7, 8]; cf. the Big Rip phenomena in cosmologies of type II, III, and IV (what is called a Nojiri–Odintsov–Tsujikawa classification of singularities is given in [9]) and is important in connection with turbulence effects [10]. Thus, the assumption about no viscous (ideal) fluid is inaccurate.

We will consider the holographic description of the universe, which implies that all information about the system parameters can be described in the form of a hologram, associated with the surface area of cosmic space [11].

The generalized holographic dark energy (HDE) model was proposed by Nojiri and Odintsov [12, 13]. Furthermore all known models of HDE are a consequence of the Nojiri–Odintsov model [14–16]. The development of various approaches and generalizations of HDE is given in various reviews [17–19]. Holographic theory is shown to be in agreement with astronomical observations [17, 20–24].

The motivation of this study is that, despite the observations tell us that the universe is essentially flat [25], it is nevertheless impossible to exclude by 100% certainty that the universe has a finite curvature; cf., for instance, the discussion in Refs. [26, 27].

Further, we will consider the Friedmann–Robertson–Walker (FRW) metric with a finite curvature of our Universe, and will investigate the consequences of this assumption for the Friedmann equation. We will obtain a holographic description of cosmological models associated with an inhomogeneous viscous dark fluid, and will discuss the singular behavior of the Universe determined by this model.

According to the holographic principle, all physical quantities in the universe including the dark energy can be described by specified values on the space-time boundary [28, 29]. The typical HDE density can be described via the Planck mass M_p and a characteristic length L_{IR} (infrared radius) [11]

$$\rho_{hol} = 3c^2 M_p^2 L_{IR}^{-2}, \tag{1}$$

where c is an arbitrary dimensionless positive parameter while the universe is expanding.

* E-mail: iver.h.brevik@ntnu.no
** E-mail: alex.timosh@rambler.ru

Let us consider the homogeneous and isotropic FRW universe in which the metric has the form

$$ds^2 = -dt^2 + a^2(t)\Big(\frac{dr^2}{1-\lambda r^2} + r^2 d\Omega^2\Big), \qquad (2)$$

where $a(t)$ is the scale factor and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the metric of the space. The quantity λ characterizes the curvature of the three-dimensional space.

As is known, the metric (2) describes a homogeneous and isotropic expanding space. We may have $\lambda = 0$ (spatially flat space), $\lambda = +1$ (closed universe), or $\lambda = -1$ (open universe). The open universe expands forever; the flat universe also expands forever but at $t \to +\infty$ the expansion occurs at constant speed; the closed universe expands to a certain instant, after which the expansion is replaced by a compression leading to a collapse.

The Friedmann equation for a one-component fluid with nonzero curvature has the form

$$H^2 = \frac{k^2}{3}\rho - \frac{\lambda}{3}, \qquad (3)$$

where ρ is the HDE density, $k^2 = 8\pi G$ with G the Newtonian gravitational constant, and $H(t) = \dot{a}(t)/a(t)$ is the Hubble function.

The infrared radius L_{IR} can be identified with size of the horizon of particles L_p or with size of the event horizon L_f [12]. However, not all ways of choosing infrared radius will lead to an accelerated expansion of the universe so that the choice of an infrared radius is not arbitrary.

The holographic energy density is known to be basically the same as the energy of the infrared (IR) radiation. If we identify the energy ρ in equation (3) with the HDE ρ_{hol} in equation (1), then we obtain the first Friedmann equation in another form:

$$H = \sqrt{\Big(\frac{c}{L_{IR}}\Big)^2 - \frac{\lambda}{a^2}}. \qquad (4)$$

Further, we will suppose that the viscous dark fluid driving the evolution of the universe, has a holographic origin.

We will investigate the cosmological models of a viscous dark fluid, obeying the inhomogeneous equation of state (EoS) in a FRW universe [5, 30]. Dissipative processes are described by the bulk viscosity in the form [31].

Let us assume that the universe is filled with a one-component fluid, obeying the energy conservation law

$$\dot{\rho} + 3H(\rho + p) = 0. \qquad (5)$$

We apply the holographic principle for cosmological models with a constant value of the thermodynamic parameter $\omega(\rho, t)) = \omega_0$ and various values of the bulk viscosity $\zeta(H, t)$, and distinguish between two cases.

Case 1. Fluid model with constant $\omega(\rho, t) = \omega_0$ and constant viscosity $\zeta(H, t) = \zeta_0$.

In this simple case the EoS will take the form

$$p = \omega_0\rho - 3\zeta_0 H. \qquad (6)$$

The solution of the energy conservation equation is

$$a(t) = \sqrt{C_1 + C_2 e^{\tilde{\zeta}_0 t} + \theta t}, \qquad (7)$$

where θ is a parameter associated with the spatial curvature, $\tilde{\zeta}_0$ is a modified viscosity parameter, and C_1, C_2 are arbitrary constants. The expression (7) for the scale factor contains the correction θt, which is associated with the spatial curvature.

For the Hubble function we will obtain

$$H(t) = \frac{1}{2}\frac{C_2\tilde{\zeta}_0 e^{\tilde{\zeta}_0 t} + \theta}{C_1 + C_2 e^{\tilde{\zeta}_0 t} + \theta t}. \qquad (8)$$

Let us analyze the Hubble function with respect to the singular behavior. If C_1 and C_2 are positive, then in a flat universe ($\theta = 0$) there is no singularity. In an open universe ($\theta < 0$) it is possible to form a singularity after a finite time span (type Big Rip [6]).

In the asymptotic limit $t \to 0$ (inflationary epoch) the Hubble function tends to the constant value

$$H(t) = \frac{C_2\tilde{\zeta}_0 + \theta}{C_2\tilde{\zeta}_0 + C_1},$$

while in the other limit $t \to \infty$ (dark energy epoch) it goes again to a constant $H(t) \to \tilde{\zeta}_0$ regardless of the spatial curvature. In both cases, the universe is in a state of accelerated expansion.

The particle horizon L_p for nonzero spatial curvature is

$$L_p = \frac{1}{\theta}\int\limits_{C_1}^{\theta t + C_1} \frac{dx}{\sqrt{\alpha e^{\beta x} + x}}, \qquad (9)$$

where $\alpha = C_2 e^{-C_1\beta}$, $\beta = \tilde{\zeta}_0/\theta$, and $\theta \neq 0$.

In the particular case when $C_1 = 0$, $C_2 = 1$ ($\alpha = 1$), we obtain in the initial stage inflation

$$L_p(t \to 0) = \frac{2}{\gamma}\sqrt{1+\gamma t}(\sqrt{1+\gamma t} - 1), \qquad (10)$$

where $\gamma = \theta + \tilde{\zeta}_0$.

The Hubble function can be expressed in terms of the particle horizon and its derivatives as [13]

$$H = \frac{\dot{L}_p - 1}{L_p}, \quad \dot{H} = \frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p^2}{L_p^2} + \frac{\dot{L}_p}{L_p^2}. \tag{11}$$

As a result, the energy conservation law can be expressed in a holographic form as

$$\frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p^2}{L_p^2} + \frac{\dot{L}_p}{L_p^2} + \tilde{\omega}_0\Big(\frac{\dot{L}_p - 1}{L_p}\Big)^2 + \\ + \big[2\lambda(\tilde{\omega}_0 + 1)(C_2\tilde{\zeta}_0 e^{\tilde{\zeta}_0 t} + \theta) - \tilde{\zeta}_0\big]\,\frac{\dot{L}_p - 1}{L_p} = 0. \tag{12}$$

Case 2. Fluid model with constant $\omega(\rho, t) = \omega_0$ and viscosity proportional to the Hubble function, $\zeta(H, t) = 3\tau H$.

In this case the EoS takes the form

$$p = \omega_0\rho - 9\tau H^2, \tag{13}$$

where τ is a positive dimensional constant.

The solution of the differential equation (5) becomes

$$a(t) = \sqrt{C_1 t + C_2 - \frac{1}{2}\,\tilde{\lambda}\tilde{\tau}t^2}, \tag{14}$$

where $\tilde{\tau}$ and $\tilde{\lambda}$ are the modified viscosity and curvature parameters and C_1, C_2 are arbitrary constants.

The Hubble function is

$$H(t) = \frac{1}{2}\,\frac{C_1 - \tilde{\lambda}\tilde{\tau}t}{C_1 t + C_2 - \frac{1}{2}\,\tilde{\lambda}\tilde{\tau}t^2}. \tag{15}$$

We analyze again this result: if $C_1 \neq 0$ and $C_2 \neq 0$ in a space with nonzero curvature, there will be two singularities of the Big Rip type, while in the case of flat space, only one is formed. Thus, the curvature of space leads to additional singularities. In the asymptotic cases of early or late universe, the Hubble function tends to a constant, independent of the curvature.

Calculation of the particle horizon in the case of nonzero curvature, when $C_1 = 0$, $C_2 = 1$, leads to the result

$$L_p = \sqrt{\frac{2}{\tilde{\lambda}\tilde{\tau}}}\sqrt{1 - \frac{1}{2}\,\tilde{\lambda}\tilde{\tau}t^2}\,\arcsin\Big(\sqrt{\frac{1}{2}\,\tilde{\lambda}\tilde{\tau}t}\Big). \tag{16}$$

The energy conservation law in holographic form is

$$\frac{\ddot{L}_p}{L_p} - \frac{\dot{L}_p^2}{L_p^2} + \frac{\dot{L}_p}{L_p^2} + \tilde{\tau}\Big(\frac{\dot{L}_p - 1}{L_p}\Big)^2 + \\ + 2\tilde{\lambda}\big(C_1 - \tilde{\tau}\tilde{\lambda}t\big)\,\frac{\dot{L}_p - 1}{L_p} = 0. \tag{17}$$

Summarizing the main results of this work, we have obtained, on the basis of the holographic principle, a description of two distinguished viscous dark fluid cosmological models, and we have discussed the singular behavior of the universe when determined by these models. We have shown that the inclusion of nonzero curvature in the Friedmann equation leads to additional singularities of type Big Rip in the Universe. The application of the holographic method has led to theoretical predictions in good agreement with astronomical observations [32].

Funding. This work was supported by Russian Fund for Fundamental Studies, Project No. 20-52-05009 (A. V. T.).

The full text of this paper is published in the English version of JETP.

REFERENCES

1. A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, et al., Astron. J. **116**, 1009 (1998).

2. S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, et al., Astrophys. J. **517**, 565 (1999).

3. K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, Astrophys. Space Sci. **342**, 155 (2012).

4. S. Nojiri and S. D. Odintsov, Phys. Rep. **505**, 59144 (2011).

5. S. Nojiri and S. D. Odintsov, Phys. Lett. B **639**, 144 (2006).

6. S. Nojiri and S. D. Odintsov, Phys. Rev. D **72**, 144 (2005).

7. I. Brevik, S. Nojiri, S. D. Odintsov, and D. Saez-Gomez, Eur. Phys. J. C **69**, 563 (2010).

8. I. Brevik, E. Elizalde, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **84**, 103508 (2011).

9. S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005).

10. I. Brevik, R. Myrzakulov, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **86**, 063007 (2012).

11. M. Li, Phys. Lett. B **603**, 1 (2004); arXiv: hep-th/0403127.

12. S. Nojiri and S. D. Odintsov, Gen. Relativ. Gravit. **38**, 1285 (2006); arXiv: hep-th/0506212.

13. S. Nojiri and S. D. Odintsov, Eur. Phys. J. C **77**, 528 (2017).

14. S. Nojiri, S. D. Odintsov, and Tanmoy Paul, Phys. Lett. B **825**, 136844 (2022).

15. S. Nojiri, S. D. Odintsov, and Tanmoy Paul, Symmetry **13**, 928 (2022).

16. S. Nojiri, S. D. Odintsov, V.K. Oikonomou, and Tanmoy Paul, Phys. Rev. D **102**, 023540 (2020).

17. K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, Astrophys. Space. Sci. **342**, 155 (2012).

18. S. Nojiri, S. D. Odintsov, and V. K. Oikonomou, Phys. Rep. **692**, 1 (2017).

19. S. Wang, Y. Wang, and M. Li, Phys. Rep. 696, 1 (2017); arXiv: 1612.00345 [astro-ph.CO]; S. Wang, Y. Wang, and M. Li, Phys. Rep. **696**, 198 (2017).

20. J. Lu, E. N. Saridakis, M. R. Setare, and L. Xu, J. Cosmol. Astrophys. **1003**, 031 (2010); arXiv: 0912.0923 [astro-ph.CO].

21. X. Zhang and F. Q. Wu, Phys. Rev. D **72**, 0435 (2005); arXiv: astro-ph/0506310.

22. M. Li, X. D. Li, S. Wang, and X. Zhang, J. Cosmol. Astrophys. **0906**, 036 (2009); arXiv: 0904.0928 [astro-ph.CO].

23. Q. G. Huang and Y. G. Gong, J. Cosmol. Astrophys. **0408**, 006 (2004); arXiv: astro-ph/0403590.

24. B. Wang, E. Abdalla, and R. K. Su, Phys. Lett. B **611**, 21 (2005); arXiv: hep-th/0404057.

25. Planck Collaboration 2019, Astron. Astrophys. **641**, A1 (2020); arXiv:1807.06205 [astro-ph.CO].

26. E. Di Valentino, A. Melchiorri, and J. Silk, Nature Astron. **4**, 196 (2020).

27. W. Handley, Phys. Rev. D **103**, L041301 (2021).

28. J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).

29. S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

30. S. Myrzakul, R. Myrzakulov, and L. Sebastiani, Astrophys. Space Sci. **350**, 845 (2014).

31. S. Capozziello, V. F. Cardone, E. Elizalde, S. Nojiri, and S. D. Odintsov, Phys. Rev. D **73**, 043512 (2006).

32. A. V. Astashenok and A. S. Tepliakov, Int. J. Mod. Phys. D **29**, 1950176 (2020); arXiv: 1911.01422v1 [gr-qc].