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On the accuracy of conductance quantization in spin-Hall insulators

S. K. Konyzheva, E. S. Tikhonov, V. S. Khrapai[1)]

Institute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Russa

Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia

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Transverse conductance in an ordinary quantum Hall effect is quantized with metrological accuracy. In the effective Landauer-Büttiker description [1] this is interpreted as a conductance quantization of one-dimensional (1D) edge channels. Such edge channels are protected by chirality thereby their four-terminal resistance vanishes. By contrast, conventional 1D systems inevitably suffer from contact effects [2] and even the best ones exhibit poor quantization [3] and non-vanishing four-terminal resistance [4].

Somewhat intermediate case is realized in quantum spin-Hall (QSH) insulators [5, 6]. Here, the electric current is carried by a pair of 1D helical edge channels with opposite spin and chirality, thereby the backscattering is easier than in the quantum Hall case and more difficult compared to the conventional 1D case. In spite of the expected immunity to a non-magnetic disorder in a phase-coherent helical edge channel [7], the mean-free path in experiments is relatively small and longer channels behave as quasi-classical diffusive conductors [8, 9]. The shorter, ballistic, channels exhibit four-terminal resistance which is poorly quantized and additive. Often the resistance randomly drops below the quantum value $g_0^{-1} = h/e^2$ in local measurements [6] and below the expected fraction of g_0^{-1} in non-local measurements. This indicates that the measured signal is not the 1D conductance and is influenced by contact effects. Backscattering of helical electrons at a contact can be revealed in transport [10] and noise measurements as well as in spin injection and photogalvanic experiments.

In this work, we elaborate the role played by the leads and ohmic contacts in resistance measurements in ballistic helical edge channels. A simple model of a phase-incoherent transport taking spin relaxation in the leads and contacts into account is presented for a realistic experimental setup. We observe that the four-terminal resistance is always below g_0^{-1} and vanishes in the absence of spin relaxation. Similarly, a non-additivity of the edge resistances is observable in a two-terminal measurement. We bridge our results with the model of disordered contacts [10] and estimate the spin relaxation resistance contribution in HgTe-based QSH devices.

To begin with, we develop an experimentally relevant model of a multi-terminal bar for QSH measurements. In a typical experimental setup [6], a lithographic gate covers the inner part of the mesa, excluding the leads. All the leads are assumed identical and are represented by regions of two-dimensional electron gas. The leads have finite resistance and interconnect helical edge channels with the ohmic contacts. The ohmic contacts have negligible resistance and serve as macroscopic equilibrium reservoirs, connecting the device to external electric circuit. We will consider the idealized case of ballistic topologically protected edge states, such that the spin relaxation occurs only in the leads and the ohmic contacts. In addition, we assume that the edge channels are perfectly coupled to the leads. This means that the chemical potentials of the outgoing edge channels coincide with those of the same-spin electrons in the leads nearby the bulk-edge transition point. All the leads are assumed to be quasi-1D, such that any dependence of the chemical potentials within the cross-section of the lead is neglected. The lead conductance is denoted g_L. The spin relaxation in the leads is taken into account via the total spin relaxation conductance as $G_{\rm s} = g_{\rm s} + g_L/4$, which takes into account two contributions, from a direct spin- relaxation ($g_{\rm s}$ term) and from an indirect relaxation via out-diffusion into the ohmic contact ($g_L/4$ term).

Our main result is the expression for a four-terminal resistance of the helical edge channel, determined as the ratio of the measured voltage drop between the ohmic contacts on either side of the channel to the current in the channel. Such resistance reads:

$$R_{4T} = \frac{1}{g_0}\left[\frac{G_{\rm s}}{G_{\rm s}+g_0}\right]. \tag{1}$$

[1)]e-mail: dick@issp.ac.ru

Equation (1) dictates that the measured R_{4T} is always smaller than the quantum resistance g_0^{-1}. In the limit of large lead resistance and negligible spin relaxation ($G_\mathrm{s} \to 0$) the R_{4T} is suppressed down to zero, similar to zeroing of the longitudinal resistance in the ordinary quantum Hall regime. Thus, apparently, the requirement of quantum phase-coherence in the leads raised in [7] is excessive as long as the spin relaxation is suppressed.

In the opposite limit $G_\mathrm{s} \gg g_0$ the spin relaxation is strong and we obtain a small correction to the resistance quantum $R_{4T} \approx g_0^{-1} - G_\mathrm{s}^{-1}$. It is interesting to compare with [10], which addresses a QSH measurement with disordered current/voltage probes. Relevant to our case, that calculation predicts $R_{4T} = g_0^{-1}(1-D)/(1+D)$, where D is a reflection probability at a contact(formula (17) with all D the same [10]). This coincides with the result (1) given $D = (1+2G_\mathrm{s}/g_0)^{-1}$, thereby bridging the model of disordered probes [10] with the spin relaxation in the leads in typical experiments.

We also calculate a two-terminal resistance between the neighboring contacts in an N-terminal bar, with the following result:

$$R_{2T} = \frac{2}{g_L} + \frac{1}{2g_0} + \frac{(N-2)(G_\mathrm{s}+2g_0)}{(NG_\mathrm{s}+2g_0)(G_\mathrm{s}+g_0)}\frac{G_\mathrm{s}}{2g_0}. \quad (2)$$

The first two terms in (2) are, respectively, the inevitable contribution of the lead resistance and the resistance of two helical edges in parallel. The last term takes a non-additivity of the helical edge resistances into account, given the spin relaxation is finite. In the limit $G_\mathrm{s} \to 0$, as well as for $N = 2$, this term vanishes and we recover a result equivalent to the ordinary (spin-degenerate) quantum Hall effect. In the opposite limit $G_\mathrm{s} \gg g_0$ the edge resistances become completely additive and R_{2T} is a sum of the lead resistance and the edge resistances g_0^{-1} and $(N-1)g_0^{-1}$ connected in parallel. In a multi-terminal bar with $N \to \infty$ we have $R_{2T} \approx 2g_L^{-1} + g_0^{-1} + (2G_\mathrm{s})^{-1}$, i.e., the first-order correction here is opposite in sign compared to the R_{4T} case.

Finally, we estimate the contribution of the spin relaxation resistance G_s^{-1} in experiments, based on the measurements of the weak anti-localization in HgTe quantum wells. We obtain, roughly, $G_\mathrm{s}^{-1} \sim 100\,\Omega$, such that the expected contribution of the spin relaxation resistance to R_{4T} and R_{2T} is within a few percent. This estimate is consistent with the rule of thumb that the edge resistances are additive in the experiments, as well as with numerous observations of R_{4T} below the quantum value $g_0^{-1} = h/e^2$ in local measurements [6] and below the expected fraction of g_0^{-1} in non-local measurements.

In summary, we have shown how the spin relaxation in the current/voltage leads affects the resistance measurements of ballistic QSH helical edge channels in experimentally relevant geometries. Negligible relaxation results in a vanishing four-terminal resistance and non-additive edge resistances in a two-terminal setup even if the quantum phase-coherence is not preserved, similar to the case of ordinary quantum Hall effect. Available experiments are in the opposite limit of strong spin relaxation, which explains a poor quality of the resistance quantization as well as the edge resistances smaller than expected ballistic value.

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