# Comment on "Noise in the helical edge channel anisotropically coupled to a local spin" (Pis'ma v ZhETF 108, 700 (2018))

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In [1] the current noise in the helical edge channel anisotropically coupled to a local spin 1/2 has been computed. In addition to the noise, a result for the backscattering current $I_{\mathrm{bs}}$ was reported. The latter formula (see Eq. (7) of [1]) does not coincide with the expression for $I_{\mathrm{bs}}$ derived in our recent work (see Eq. (22) of [2]) for a general form of the exchange interaction matrix. Below we shall argue that, in general, the result of [1] for the backscattering current is *erroneous*. Equation (7) of [1] gives the correct answer for the diagonal exchange matrix only. The incorrect result of [1] is a consequence of the assumption (which was also done in [3]) that the density matrix of the impurity spin, $\rho_S$, is diagonal in the eigenbasis of $S_z$ (see Eq. (2) of [1]). As we demonstrated in [2], a careful analysis of the problem invalidates this assumption.

In order to set notations, we define the Hamiltonian describing the exchange interaction between the helical edge states and a magnetic impurity as $H_{\mathrm{int}} = J_{jk} S_j s_k$, where $\mathbf{S}$ ($\mathbf{s}$) denotes the operator of the impurity spin (the spin density of helical electrons) and $J_{jk}$ is a $3\times 3$ exchange matrix. In [1] the following form of the exchange matrix was considered

$$J = \begin{pmatrix} 2(J_0+J_2) & 0 & 2J_a \\ 0 & 2(J_0-J_2) & 0 \\ 2J_1 & 0 & J_z \end{pmatrix}. \qquad (1)$$

We note that in our paper [2] we used dimensionless exchange matrix $\mathcal{J}_{jk} = \nu J_{jk}$. Here $\nu = 1/(2\pi v)$ stands for the density of states per edge mode and $v$ denotes the velocity of the helical states.

To illustrate our point we first consider the case $J_2 = J_1 = 0$ and the regime $V \gg T$. Then, according to Eq. (7) of [1] the backscattering current is given by ($G_0 = e^2/h$)

$$I_{\mathrm{bs}}^{\mathrm{NRS}} = -G_0 T J_a^2/(2v^2). \qquad (2)$$

This result should be contrasted with our result [2]:

$$I_{\mathrm{bs}} = -G_0 \frac{V}{2v^2} \frac{2J_a^2 J_0^2}{2J_a^2 + J_z^2}. \qquad (3)$$

In addition to a very different dependence of the backscattering current on the elements of the exchange matrix, Eq. (2) predicts saturation of the backscattering current at $V \gg T$ whereas Eq. (3) does not. This saturation occurs due to the full polarization of the magnetic impurity along $z$-axis by the applied voltage $V \gg T$. However, such a polarization is a consequence of an erroneous assumption that $\rho_S$ is diagonal in the eigenbasis of $S_z$. In fact, there are no physical reasons for the full polarization (along $z$-axis) to occur: the magnetic impurity remains partially polarized in a direction tilted with respect to $z$-axis for arbitrary large voltage (see discussion around Eq. (26) in [2]).

To be more specific, the polarization along $z$-axis predicted by [1] follows from a claim that the dephasing of the impurity spin is mainly induced by the term $J_z S_z s_z$ in $H_{\mathrm{int}}$. However, the term $2J_a S_x s_z$ enters $H_{\mathrm{int}}$ on the equal grounds and thus has to be taken into consideration to properly account for the dephasing. In particular, if $J_z = 0$ the magnetic impurity gets polarized along $x$-axis for $V \gg T$. In this regime, the backscattering is induced by the term $2J_0(S_x s_x + S_y s_y)$ in the Hamiltonian and is insensitive to the precise value of $J_a$. This is consistent with our Eq. (3) and not consistent with Eq. (2).

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Secondly, we consider the case $J_2 = J_a = 0$. Then, Eq. (7) of [1] predicts a linear in $V$ backscattering current

$$I_{\text{bs}}^{\text{NRS}} = -G_0 \frac{V}{4v^2} J_1^2. \tag{4}$$

Our result for this case coincides with Eq. (4) in the regime $V \gg T$. This occurs because the density matrix of the magnetic impurity $\rho_S$ is *indeed diagonal* in the eigenbasis of $S_z$ for $J_a = 0$ and $V \gg T$.

In the regime of linear conductance ($V \ll \nu |J_{jk}| T$), our result for the backscattering current reads

$$I_{\text{bs}} = -G_0 \frac{V}{4v^2} \frac{J_1^2 (J_z^2 + 2J_1^2)}{J_z^2 + 2J_1^2 + 4J_0^2}. \tag{5}$$

The discrepancy between Eqs. (4) and (5) is due to the non-diagonal structure of $\rho_S$ in the eigenbasis of $S_z$ in the linear regime. As one can see, our result (5) transforms into Eq. (4) provided $|J_z| \gg |J_{0,1}|$, i.e., precisely when $\rho_S$ is diagonal in the eigenbasis of $S_z$.

To summarize, the result for the backscattering current reported in [1] is incorrect since its derivation relies on the erroneous assumption. This also questions the result of [1] for the current noise (for the correct result for the shot noise in the regime $V \gg T$ see [4]).

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