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GLSM for Berglund–Hübsch Type Calabi–Yau manifolds

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In this note we briefly present the results of our computation of special Kähler geometry for polynomial deformations of Berglund–Hübsch type Calabi–Yau manifolds. We also build mirror symmetric Gauge Linear Sigma Model and check that its partition function computed by Supersymmetric localization coincides with exponent of the Kähler potential of the special metric.

Special Kähler geometry is the geometrical structure underlying particular coupling constants in superstring compactifications. Knowledge of this metric is important for phenomenological questions arising in superstring theories.

A polynomial $W_0(x_1,\ldots,x_N)=\sum_{i=1}^{N}\prod_{j=1}^{N}x_j^{M_{ij}}$ in N variables is called an invertible singularity if M is an invertible matrix with positive integer coefficients.

We explicitely describe the Kähler geometry on the space of polynomial deformations of $W_0(x)$ which corresponds to the space of deformations of complex structures on the Calabi–Yau manifold X_0. The exponent of the Kähler potential of the Weil–Peterson metric on the deformation space is expressed in terms of the periods the periods $\sigma_a(\phi)$ as follows:

$$e^{-K(\phi,\bar{\phi})}= \tag{1}$$

$$=\sum_{a=0,2d-1}(-1)^{|a|/d}\prod_{i\le 5}{}^{'}\frac{\Gamma\left((a_j+1)M_{ji}^{-1}\right)}{\Gamma\left(1-(a_j+1)M_{ji}^{-1}\right)}|\sigma_a(\phi)|^2.$$

The partition function for general Gauge Linear Sigma Models (GLSM) was computed exactly a few years ago. The conjecture of Jockers et al. states that this partition function of GLSM coincides with the exponent of the Kähler potential of the metric on the Kähler moduli space of the infrared limiting model of the GLSM in the Calabi–Yau case.

Mirror symmetry connects quantum corrected Kähler moduli space of one model with complex structures moduli space of another. For all Calabi–Yau threefolds given by invertible singularities we construct the corresponding mirror symmetric GLSM and compute their partition functions. Then we explicitly check that they coincide with exponents of Kähler potentials of the metrics on the deformation spaces of the invertible singularities.

Namely, we use Batyrev approach to the mirror symmetry. The idea is as follows. Let Calabi–Yau threefold X be defined as a hypersurface in a weighted projective space $\mathbb{P}^4_{(k_1,\ldots,k_5)}$ and given by zero locus of the polynomial $W(x,\phi)$. Exponents of the monomials in the deformed polynomial $W(x,\phi)=\sum_{i=1}^{5}\prod_{j=1}^{5}x_j^{M_{ij}}+\sum_{l=1}^{h}\phi_l\prod_{j=1}^{5}x_j^{s_{lj}}$ define the finite set $\mathbf{V}_a$, $a=1,\ldots,N$ and thus define Batyrev's polytope Δ_X. Knowing the set of vectors $\mathbf{V}_a$ we can construct a fan, which defines another toric variety. Calabi–Yau manifold Y, the mirror to X, can be defined as a hypersurface in this variety given by the zero locus of the homogeneous polynomial W_Y. Using the fan we also find the corresponding GLSM with its gauge group and the charges Q_{al} of its chiral multiplets. The charges appear as coefficients of linear relations between the vectors of the fan and set the weights of the toric variety.

The mirror symmetric GLSM has $h+5$ chiral superfields fields $\{\Phi_a\}_{a=1}^{h+5}$ and h $U(1)$ vector superfields $\{V_l\}_{l=1}^{h}$ acting on Φ_a with the charge matrix $\{Q_{al}\}_{a\le h+5,l\le h}$. In our matrix $Q_{al}=s_{lj}M_{ja}^{-1}$ if $\le a\le 5$ and $Q_{al}=-\delta_{a-5,l}$ if $a>5$.

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Then the general expression for the partition function specifies in our case to

$$Z_Y = \sum_{\bar{m}\in\Lambda} \int_{\mathcal{C}} \prod_{l=1}^{h} \frac{d\tau_l}{(2\pi i)} \left(z_l^{-p_l} \bar{z}_l^{-\bar{p}_l}\right) \frac{\Gamma(1-p_h)}{\Gamma(\bar{p}_h)} \times$$

$$\times \prod_{a=1}^{5} \frac{\Gamma\big(\sum_l p_l\, s_{lj} M_{ji}^{-1}\big)}{\Gamma\big(1-\sum_l \bar{p}_l\, s_{lj} M_{ji}^{-1}\big)} \prod_{l=1}^{h-1} \frac{\Gamma(-p_l)}{\Gamma(1+\bar{p}_l)}. \qquad (2)$$

In this setting we compute the integral via residues and reproduce the formula (1) up to rescaling and simple coordinate change between Fayet–Iliopoulos parameters of the GLSM and complex deformation parameters $\{\phi_l\}_{l=1}^{h}$, which is the mirror map.

Thus, starting from the model of Berglund–Hübsch type Calabi–Yau manifolds X we have constructed the $\mathcal{N}=(2,2)$ Gauged Linear Sigma Model with the manifold of supersymmetric vacua Y, which is the mirror for X. Having computed the Special geometry on the moduli space of complex structures on X and using Batyrev's approach to Mirror symmetry we have checked JKLMR conjecture.

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