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On thermal Nieh–Yan anomaly in topological Weyl materials

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We discuss the possibility of a gravitional Nieh–Yan anomaly of the type $\partial_\mu j^\mu_5 = \gamma T^2 \mathcal{T}^a \wedge \mathcal{T}_a$ in topological Weyl materials, where T is temperature and $\mathcal{T}^a$ is the effective or emergent torsion. As distinct from the non-universal parameter Λ in the conventional (zero temperature) Nieh–Yan (NY) anomaly [1–4] – with canonical dimensions of momentum – the parameter γ is dimensionless. This suggests that the dimensionless parameter is fundamental, being determined by the geometry, topology and the number of chiral quantum fields without any explicit non-universal ultraviolet (UV) scales.

In non-relativistic topological matter, quasi-relativistic description of low-energy quasiparticles with linear spectrum phenomena may emerge [5, 6]. In three spatial dimensions at a generic (two-fold) fermion band crossing at momentum $\mathbf{p}_W$, the Hamiltonian is of the Weyl form [5, 7–9] $H_W = \sigma^a e^i_a (p - p_W)_i + \cdots$, where the e^i_a are the linear coefficients of the Pauli matrices σ^a, playing the role of background spacetime tetrad fields. The shift of the Weyl node $\mathbf{p}_W$ acts as an emergent (axial) gauge field. These background fields imply the chiral anomaly for the low-energy massless quasiparticles, see, e.g., [5, 9–11]. In particular, the non-trivial coordinate dependence (torsion) related to the tetrads $e^\mu_a(x)$ can lead to the gravitational NY anomaly [1–4, 12, 13]. Here we discuss this anomaly in the presence of finite temperature [14, 15].

For spacetimes with torsion $\mathcal{T}$ (and curvature R) the 4-dimensional invariant was introduced [1, 2, 4]:

$$N = \mathcal{T}^a \wedge \mathcal{T}_a - e^a \wedge e^b \wedge R_{ab} \tag{1}$$

and can be associated with a difference of two topological terms [3]. It was also suggested that N contributes to the anomalous chiral current j^μ_5:

$$\partial_\mu j^\mu_5 = \frac{\Lambda^2}{4\pi^2} N(\mathbf{r}, t), \tag{2}$$

where the parameter Λ has dimension of mass $[\Lambda] = [M]$ and is determined by an UV scale.

There has been several attempts to consider the NY anomaly in condensed matter systems with Weyl fermions, see, e.g., [13, 16–18]. However, in non-relativistic systems the relativistic high-energy cut-off Λ is not a well defined parameter. The complete UV theory is non-Lorentz invariant and the linear, quasirelativistic Weyl regime is valid at much lower scales. Moreover, the anomalous hydrodynamics of superfluid ^{3}He at zero temperature suggests that the chiral anomaly is completely exhausted by the emergent axial gauge field corresponding to the shift of the node or, conversely, the NY anomaly term. Nevertheless, it was shown in [13] that the low-energy theory satisfies the symmetries and conservations laws related to an emergent quasirelativistic spacetime with torsion and Λ is determined from the UV-scale where the linear Weyl approximation breaks down.

The fully relativistic responses work unambiguously only for terms in the effective action with dimensionless coefficients. An example is the 2+1-dimensional topological Chern–Simons (CS) terms describing the quantum Hall effect. Gravitational CS terms similarly are quantized in terms of chiral central charge which has relation to thermal transport and the gapless boundary modes [19, 20]. The CS action was recently generalized to crystalline topological insulators in odd space dimensions. The CS term is expressed via elasticity tetrads E with dimension $[E] = [M]$ as the topological term $E \wedge A \wedge dA$ with quantized dimensionless coefficients [21–23].

Another such example is the temperature correction to curvature effects, with $\delta S_{\rm eff} = \int T^2 \mathcal{R}$ in the low-energy action [24]. This represent the analog of the gravitational coupling (Newton constant) in the low-energy action where the curvature scalar $\mathcal{R}$ is some analog of scalar spacetime curvature. Since $[T]^2[\mathcal{R}] = [M]^4$, the coefficient of this term in the low-energy theory is dimensionless, and thus can be given in terms of universal constants. It is fully determined by the number of the fermionic and bosonic species and works both in relativistic and non-relativistic systems [24].

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The same universal behavior takes place with the terms describing the chiral magnetic and chiral vortical effects in Weyl superfluids, where the coefficients are dimensionless [5, 25]. Similarly, the coefficient of the $R\wedge R$ gravitional anomaly in chiral Weyl systems affects the thermal transport coefficients in flat space [26, 27]. These coefficients are fundamental, being determined by the underlying degrees of freedom in addition to symmetry, topology and geometry. The same may hold for the temperature correction to the NY anomaly term. The zero-temperature anomaly term is still not confirmed in general. On one hand the UV cut-off parameter Λ is not well-defined in relativistic field theory. On the other hand, such a cut-off is not in general available in non-relativistic matter and can be anisotropic [13] or even zero. However, the term $T^2(\mathcal{T}^a \wedge \mathcal{T}_a + e^a \wedge e^b \wedge R_{ab})$ has the proper dimensionality.

Here we use the result obtained in [28, 29] for the finite temperature contribution to the chiral current. For complex fermions, the chiral current is $j_5^k = -\frac{T^2}{24}\epsilon^{0kij}T^0_{ij}$, which can be covariantly generalized to the 4-current $j_5^\mu = -\frac{T^2}{24}\epsilon^{\mu\nu\alpha\beta}e_{\nu a}T^a_{\alpha\beta}$ leading to $\partial_\mu j_5^\mu = -\frac{T^2}{48}\epsilon^{\mu\nu\alpha\beta}T_{a\mu\nu}T^a_{\alpha\beta}$. With curvature, this becomes the temperature correction to the full NY term, where the cut-off Λ is substituted by the well defined local relativistic temperature T with parameter $\gamma = 1/12$:

$$\partial_\mu j_5^\mu = -\frac{T^2}{12}N(\mathbf{r}, t). \tag{3}$$

This temperature correction to the NY anomaly can be universal for chiral Weyl fermions. It is fully determined by the quasirelativistic physics in the vicinity of the Weyl node, and does not depend on the non-universal cut-off as distinct from the $T = 0$ term. The prefactor has been found using a relativistic regularization scheme (see [14, 28, 29]) and has been confirmed in [15] in a finite T spectral flow calculation in a torsional magnetic field [16, 30].

It is known that the hydrodynamics of ^{3}He-A experiences the chiral anomaly due to non-trivial texture [5, 30]. The spectral flow of momentum depends only on the density of states at the node. Therefore the relativistically invariant calculation near the gap node gives the same result as the full BCS Fermi-liquid far from the nodes, where relativistic invariance is lost. The same ultraviolet-infrared correspondence may take place for the finite temperature NY anomaly. The temperature is an infrared energy scale at which the quasi-relativistic fermions are well-defined, whereas Λ^2 is an explicit and non-universal UV cut-off [13]. This correspondence can be verified from the hydrodynamic conservation laws, which exactly correspond to the NY anomaly. The first attempt was made in [14], where the hydrodynamic transport parameters calculated in [31] have been used.

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