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Optical properties of $p_x + ip_y$ superconductors with strong impurities

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Observation of the polar Kerr effect in Sr_2RuO_4 [1] has lead to theoretical studies of the Hall response $\sigma_{xy}(\omega)$ in $p_x + ip_y$-wave superconductors [2–9]. A nonzero σ_{xy} implied by the Kerr effect requires either a multi-band structure or impurities in the superconductor. So far only weak impurities have been considered. We generalize existing theory to strong impurities, considering their effect on the spectrum of the chiral superconductor and on its Hall response in the limit of strong, point-like impurities of low concentration $n_{\rm imp}$.

Point-like impurities are parametrized by their scattering phase δ in the s-channel. It enters the T matrix, which describes scattering off a single impurity in a clean metal to all orders in its potential,

$$T_0 = \frac{1}{\pi\nu(i\mathrm{sign}\,\mathrm{Im}\,E - \tau_3\cot\delta)}, \qquad (1)$$

where E is energy, ν is metallic density of state and τ_3 acts in Nambu space. To describe a disordered superconducting state, we write down the Dyson equation

$$\bar{G}^{-1} = G_S^{-1} - n_{\rm imp}\bar{T}, \qquad (2)$$

where $G_S = (E - \tau_3\xi(p) - \check{\Delta})^{-1}$ is the clean superconducting Green's function with $\check{\Delta} = \Delta(\tau_1 p_x - \tau_2 p_y)/|p|$ being the $p_x + ip_y$ order parameter. $\bar{T}$ is a T matrix describing scattering in the disordered superconducting state. It is related to T_0 via

$$\bar{T} - T_0 = \bar{T}(\bar{G} - G_0)T_0, \qquad (3)$$

where $G_0 = (E - \tau_3\xi)^{-1}$ is the clean normal-state Green's function. Together, Eqs. (2), (3) form a self-consistent equation set on the disorder-averaged objects $\bar{G}, \bar{T}$. Solving these, we find that $\bar{G}(E)$ exhibits a short cut at energies close to $E_0 = \Delta\cos\delta$, where Δ is the superconducting gap. This indicates a sub-gap impurity band consisting of Yu–Shiba–Rusinov (YSR) type bound states hosted by the impurities [10–12]. We derive a semi-circle-shaped density of states centered at E_0 with the bandwidth $w = \sqrt{8n_{\rm imp}\Delta|\sin^3\delta|/\pi\nu}$. This is valid in the limit of a well-resolved band, $w \ll \ll E_0, \Delta - E_0$.

Equations (2), (3) also describe smoothing of the Bardeen–Cooper–Schrieffer cusp in the density of states at energies close to Δ. This happens on the energy scale $n_{\rm imp}^2/\Delta\nu^2$ which is parametrically smaller than the YSR bandwidth w.

The density of states resulting from Eqs. (2), (3) is illustrated by Fig. 1.

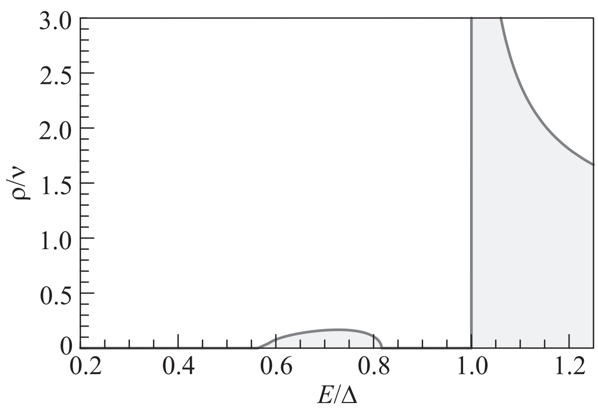

Fig. 1. (Color online) Density of states of the superconductor with strong point-like impurities at $\cos\delta = 0.7$, $n_{\rm imp} = 0.06\Delta\nu$. The semi-circle shape of the YSR impurity band is distorted due to vicinity to the continuum

Having established the effects of strong impurities on the spectrum, we turn to the calculation of $\sigma_{xy}(\omega)$. The leading contribution in small $n_{\rm imp}$ is given by the diagram on the inset of Fig. 2. It contains all orders of scattering off a single impurity, with the shaded triangles representing T matrices. Calculating the diagram, we arrive at

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$$\sigma_{xy}(\omega) = \frac{n_{\rm imp}e^2v^2\Delta\sin^3\delta}{4\omega^4}\times$$
$$\times\Big[R(E_0+\omega+i0)+R(E_0-\omega-i0)-2R(E_0)\Big], \quad (4)$$

where $R(x)$ is a cumbersome integral (see full text) depending on the ratios of x and the three energy parameters E_0, Δ, and temperature T. The function $R(x)$ is real for $|x| < \Delta$ but acquires an imaginary part at $|x| > \Delta$. This leads to threshold behavior in $\sigma_{xy}(\omega)$ as seen on Fig. 2. As ω is increased, an imaginary part in σ_{xy} first appears at $\omega = \Delta - E_0$. This threshold corresponds to processes where occupied YSR states are ionized into the continuum, and hence this process is suppressed as $\sim \exp(-E_0/T)$ at low temperatures $T \ll E_0$. The second threshold appears at $\omega = \Delta + E_0$ and corresponds to Cooper pairs being optically excited into a pair of YSR quasiparticle and continuum quasiparticle. This process is not thermally activated and therefore survives at $T \to 0$.

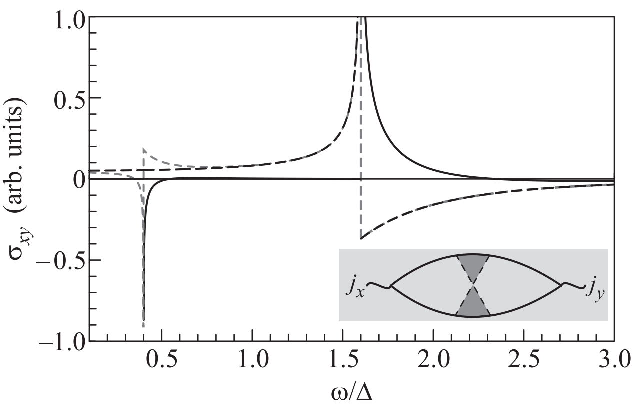

Fig. 2. (Color online) $\mathrm{Im}\sigma_{xy}(\omega)$ (solid lines) and $\mathrm{Re}\sigma_{xy}(\omega)$ (dashed lines) in a.u. at $\Delta = 1$, $E_0 = 0.6$. Black lines are at $T = 0$ while red lines are at $T = 0.1$. Inset: diagram representing $\sigma_{xy}(\omega)$ in the linear order in $n_{\rm imp}$

In the next order in $n_{\rm imp}$ a signal should also appear at $2E_0 - w < \omega < 2E_0 + w$ corresponding to the excitation of a pair of YSR quasiparticles. Furthermore we note that our calculation breaks down at $\omega < w$ where transitions within the YSR band become important and the whole ladder diagram must be taken into account. However, the latter process is thermally activated, therefore at low temperatures our result is relevant up to exponentially small $\omega \sim \exp(-\#E_0/T)$. Note that $\sigma_{xy}(\omega)$ shows no features at $\omega = 2\Delta$ indicating that processes where two continuum quasiparticles are created are irrelevant to the Hall response.

At zero temperature, the function $R(x)$ simplifies to

$$R(x\pm i0) = (2\Delta^2 - E_0x - x^2)\times$$
$$\times\begin{cases} \dfrac{\operatorname{arccos}\frac{x}{\Delta}}{\pi\sqrt{\Delta^2-x^2}}, & |x|<\Delta, \\ \dfrac{\operatorname{arccosh}\frac{x}{\Delta}}{\pi\sqrt{x^2-\Delta^2}}, & x>\Delta, \\ \dfrac{\mp i\pi - \operatorname{arccosh}\left|\frac{x}{\Delta}\right|}{\pi\sqrt{x^2-\Delta^2}}, & x<-\Delta. \end{cases} \quad (5)$$

Only the threshold process at $\omega = \Delta + E_0$ survives at $T = 0$, as seen on Fig. 2.

Experiment [1] was limited to $\omega \sim 1$ eV. In this limit, $\omega \gg \Delta, T$ we obtain

$$\sigma_{xy}(\omega) = -\frac{n_{\rm imp}e^2v^2\Delta\sin^3\delta}{4\omega^3}\times$$
$$\times\left[i\tanh\frac{E_0}{2T} + \frac{4E_0}{\pi\omega}\ln\frac{\omega}{\max\{\Delta, T\}}\right]. \quad (6)$$

In the low-T, weak impurity limit, $T \ll \Delta, E_0 \to \Delta$, this reproduces [7] up to parameters used to describe the impurities.

In conclusion, we have considered strong point-like impurities in $p_x + ip_y$ superconductors. We have shown such impurites to produce a YSR-type impurity band at $E_0 = \Delta\cos\delta$. We have calculated the anomalous Hall response σ_{xy} as a function of temperature and frequency. At high frequencies $\omega \gg \Delta, T$ its behavior is similar to that of weak impurities, while behavior at $\omega \sim \Delta$ is much richer, exhibiting thresholds at $\omega = \Delta - E_0$ and $\omega = \Delta + E_0$. The first corresponds to the onset of ionization of YSR states into the continuum and is thermally activated while the second involves creation of continuum + YSR quasiparticle pairs and survives at $T = 0$.

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