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Chiral estimate of QCD pseudocritical line

K. Zarembo[1)]

Nordita, Kungliga Tekniska Högskolan Royal Institute of Technology and Stockholm University, SE-106 91 Stockholm, Sweden

Department of Physics and Astronomy, Uppsala University, SE-751 08 Uppsala, Sweden

Institute of Theoretical and Experimental Physics, 117218 Moscow, Russia

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Spontaneously broken chiral symmetry determines many properties of hadrons and interactions among them. Chiral symmetry restoration at finite temperature causes a sharp crossover, which would be a real phase transition for zero or very light quark masses. It is believed that the crossover becomes sharper at larger baryon densities and eventually merges onto a line of first-order phase transitions at a critical endpoint [1].

Transition temperature is known quite reliably from lattice simulations: $T_0 \simeq 157$ Mev [2]. Its proximity to the mass of the π-meson suggests that the transition may be well described within the low-energy effective theory, starting with the chiral Lagrangian:

$$\mathcal{L}_\chi = \frac{F^2}{4}\,\mathrm{tr}\,\partial_\mu U^\dagger \partial^\mu U, \qquad U = \mathrm{e}^{\,iT^a\pi_a}. \tag{1}$$

This simple assumption allows one to calculate the shape of the critical line in the $T-\mu$ plane [3]. The crossover in this scenario proceeds via disordering of the chiral condensate's phase, while its modulus remains finite. Indeed, the pion field can be regarded as the phase of the condensate: $\bar{\psi}\psi \sim \Sigma\,\mathrm{e}^{\,iT^a\pi_a}$, and strong pion fluctuations, such that $\left\langle \mathrm{tr}\,\mathrm{e}^{\,iT^a\pi_a}\right\rangle = 0$, will restore chiral symmetry even if modulus is non-zero. The chiral disorder can be triggered by proliferation of skyrmions [4, 5], in parallel to the Berezinskii–Kosterlitz–Thouless transition in 2d, or by a different physical mechanism [6–9]. But no matter what the mechanism is, the transition temperature will be proportional to the pion decay constant

$$T_c = \kappa F, \tag{2}$$

just by dimensional analysis. From $T_0 = 157$ Mev and $F_\pi = 92$ Mev one finds the proportionality constant: $\kappa = 1.71$. It is interesting to note that the mean-field calculation at $N_f = 2$ gives $\kappa = \sqrt{3} = 1.73$ [7], and albeit such a perfect agreement is likely a numerical coincidence, the numbers are not an order of magnitude apart.

Taking (2) as a criterium for the phase transition, one finds for the critical line:

$$\frac{F^2(T_c(\mu),\mu)}{F^2(T_0,0)} = \frac{T_c^2(\mu)}{T_0^2}. \tag{3}$$

We will calculate the dependence of the pion decay constant on temperature and chemical potential from a simplified phenomenological model of constituent quarks interacting with the Goldstone bosons [10–13]:

$$\mathcal{L} = \bar{\psi}\left(\partial\!\!\!/ + M\ \mathrm{e}^{\,i\gamma^5 T^a\pi_a}\right)\psi. \tag{4}$$

The only parameter of the model, the constituent quark mass, can be estimated as a half of the ρ-meson mass or a third of the nucleon mass: $M = 350^{+40}_{-40}$ Mev. Simple as it is, the model yields reasonable values for low-energy constants of chiral perturbation theory [14] and underlies a quantitatively consistent description of the nucleon [12, 13].

The chiral Lagrangian arises upon integrating out quarks with subsequent derivative expansion. The pion decay constant is given by

$$\begin{aligned} F^2 &= 4N_cM^2\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{\left(p^2+M^2\right)^2} = \\ &= F_\pi^2 - \frac{N_cM^2}{2\pi^2}\int_M^\infty\frac{d\omega}{\sqrt{\omega^2-M^2}}\times \\ &\times\left(\frac{1}{\mathrm{e}^{\frac{\omega-\mu}{T}}+1}+\frac{1}{\mathrm{e}^{\frac{\omega+\mu}{T}}+1}\right). \end{aligned} \tag{5}$$

The critical line poredicted by (3), (5) is shown in Fig. 1. Its flatness at $\mu \lesssim 100$ Mev is due to the $\mathcal{O}(\mathrm{e}^{\,-M/T_0}) \simeq \simeq 0.1$ Boltzmann suppression, in quantitative agreement with the results of lattice simulations. The wiggle in the critical line near the endpoint $\mu_* = 387$ Mev follows

[1)]e-mail: zarembo@nordita.org

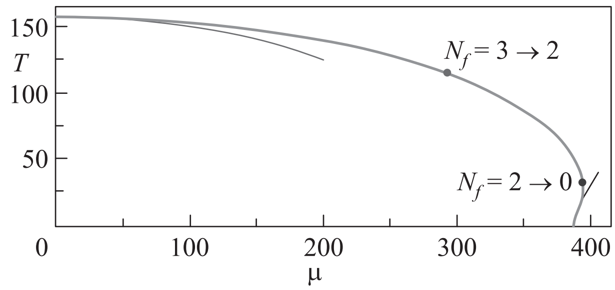

Fig. 1. (Color online) The pseudocritical line in the $T-\mu$ plane calculated from (3), (5). Boltzmann approximation and approximation of nearly degenerate Fermi gas are shown in thin lines. The dots indicate decoupling of kaons ($N_f = 3 \to 2$) and pions ($N_f = 2 \to 0$)

from the Fermi degeneracy of the quark spectrum at low temperature and chemical potentials exceeding the constituent quark mass.

The current quark masses make Goldstone fields massive:

$$F^2 m_{\text{eff}}^2 = 8N_c M m_q \int \frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+M^2} =$$
$$= F_\pi^2 m_G^2 - \frac{2N_c M m_q}{\pi^2}\int_M^\infty d\omega\,\sqrt{\omega^2-m^2}\,\times$$
$$\times\left(\frac{1}{\mathrm{e}^{\frac{\omega-\mu}{T}}+1}+\frac{1}{\mathrm{e}^{\frac{\omega+\mu}{T}}+1}\right). \tag{6}$$

The eigenvalues of the mass matrix are obtained by substituting $m_q = m_u + m_d$ for π, $m_q = m_s + m_u$ for $K^\pm$, $m_q = m_s + m_d$ for K^0, and $m_q = (4m_s + m_d + m_u)/3$ for η. Careful analysis shows that the Goldstone masses grow along the critical line, and eventually kaons and later pions decouple, becoming indistinguishable from heavier mesons. The decoupling points are indicated in Fig. 1.

The crossover line in this scenario does not have a critical endpoint, chiral symmetry restoration always proceeds without a sharp transition. Moreover the crossover becomes less and less pronounced with growing chemical potential, in contradistinction with more conventional scenarios of chiral symmetry restoration. Absence of the critical endpoind and softening of the transition at higher baryon densities are probably the most interesting qualitative features of the model considered in this paper.

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