Pis'ma v ZhETF, vol. 110, iss. 3, pp. 163 – 164

Layered superconductor in a magnetic field: breakdown of the effective masses model

A. G. Lebed[1)]

Department of Physics, University of Arizona, 1118 E. 4-th Street, AZ 85721 Tucson, USA

L. D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia

Submitted 14 June 2019
Resubmitted 14 June 2019
Accepted 15 June 2019

DOI: 10.1134/S0370274X19150049

The upper critical magnetic field, $H_{c2}(T)$, is known to be one of the most important properties of the type-II superconductors. It destroys superconductivity due to the orbital Meissner currents in case, where we can disregard the Pauli spin-splitting paramagnetic effects. The Ginzburg–Landau (GL) theory gave tools to calculate a slope of the $H_{c2}(T)$ [1] in the vicinity of superconducting transition temperature, $(T_c - T)/T_c \ll 1$. On the other hand, at zero temperature, the upper critical magnetic field was calculated for an isotropic $3D$ superconductor in [2]. Temperature dependence of $H_{c2}(T)$ in a whole temperature region in an isotropic $3D$ superconductor was calculated later in [3]. Important generalization of the GL theory to the case of anisotropic superconductors was obtained in [4], where the so-called effective mass model was implicitly introduced. The effective mass model, partially based on the results obtained in [4] in the GL region, states more: ratios of the upper critical magnetic fields measured along fixed different directions do not much depend on temperature. Recently observed experimental temperature dependencies of anisotropy of the upper critical fields in layered compound MB_2 [5] and other materials are prescribed exclusively to many-band effects (see introductory part of review [6]).

The goal of our Letter is to consider the orbital effect in a parallel magnetic field in a Q2D conductor at zero temperature, where we explicitly take into account a Q2D anisotropy of the electron spectrum. In contrast to [1–4, 6], we demonstrate that, in a Q2D case in a parallel magnetic field, the solution of the so-called gap equation can not be expressed as some exponential function. Moreover, we show that the above mentioned solution even changes a sign with changing space coordinate. This leads to unusual value of the corresponding coefficient, 0.75, in the equation,

$$H^{\parallel}_{c2}(0) \approx 0.75\ |dH^{\parallel}_{c2}/dT|_{T_c} T_c, \tag{1}$$

for a parallel magnetic field. We recall that, for a perpendicular magnetic field the corresponding solution is exponential one and gives much smaller coefficient – 0.59, as shown by different method in [7]:

$$H^{\perp}_{c2}(0) \approx 0.59\ |dH^{\perp}_{c2}/dT|_{T_c} T_c. \tag{2}$$

Note that Eqs. (1) and (2) directly break the effective mass model, since the corresponding coefficients, 0.75 and 0.59 are not close to each other.

In the Letter, we consider a layered superconductor with the following realistic Q2D electron spectrum:

$$\epsilon(\mathbf{p}) = \tfrac{1}{2m}(p_x^2 + p_y^2) - 2t_\perp \cos(p_z c^*),$$
$$t_\perp \ll \epsilon_F, \quad \epsilon_F = \tfrac{p_F^2}{2m} = \tfrac{m v_F^2}{2}, \tag{3}$$

where m – the electron in-plane mass, $t_\perp$ – the integral of overlapping of electron wave functions in a perpendicular to the conducting planes direction; ϵ_F, p_F, and v_F are the Fermi energy, Fermi momentum, and Fermi velocity, correspondingly; $\hbar \equiv 1$. In a parallel to the conducting planes magnetic field,

$$\mathbf{H} = (0, H, 0)\ , \quad \mathbf{A} = (0, 0, -Hx), \tag{4}$$

we make use of the so-called Peierls substitution method:

$$p_x \to -i\left(\frac{\partial}{\partial x}\right), \quad p_y \to -i\left(\frac{\partial}{\partial y}\right),$$
$$c^* p_z \to -ic^*\left(\frac{\partial}{\partial z}\right) - \left(\frac{\omega_c}{v_F}\right)x, \quad \omega_c(H) = \frac{e v_F c^* H}{c}. \tag{5}$$

Under such conditions the electron orbital Hamiltonian in a magnetic field can be written in the following way:

$$\hat{H} = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - 2t_\perp \cos\left(-ic^*\frac{\partial}{\partial z} - \frac{\omega_c}{v_F}x\right). \tag{6}$$

[1)]e-mail: lebed@email.arizona.edu

Using the standard procedure [2], we linearize the Hamiltonian (6) with respect to the derivative $\frac{\partial}{\partial x}$. As a result, the problem becomes exactly solvable (see, for example, [8]). It is straightforward to find the Matsubara's Green's functions [8, 9] in the magnetic field (4) and, then, to find the following Gor'kov's gap equation, which determines the parallel upper critical magnetic field at any temperature:

$$\Delta(x) = g\Big\langle \int_d^\infty \frac{2\pi T dz}{v_F \sinh\left(\frac{2\pi T z}{v_F}\right)} \times$$
$$\times J_0\left\{\frac{2t_\perp \omega_c}{v_F^2}[z(2x + z\sin\alpha)]\right\} \Delta(x + z\sin\alpha)\Big\rangle_\alpha, \quad (7)$$

where $\Delta(x)$ is the so-called superconducting gap, g is the coupling constant, d is the cut-off distance; $< ... >_\alpha$ stands for averaging procedure over angle α.

We recall that Eq. (7) determines the parallel upper critical field at any temperatures. It is possible to prove that near transition temperature, $\tau = \frac{T_c - T}{T_c} \ll 1$, it reduces to differential GL equation:

$$-\xi_\parallel^2 \frac{d^2\Delta(x)}{dx^2} + \left(\frac{2\pi H}{\phi_0}\right)^2 \xi_\perp^2 x^2 \Delta(x) - \tau\Delta(x) = 0, \quad (8)$$

$$\xi_\parallel = \frac{\sqrt{7\zeta(3)} v_F}{4\sqrt{2}\pi T_c}, \quad \xi_\perp = \frac{\sqrt{7\zeta(3)} t_\perp c^*}{2\sqrt{2}\pi T_c}, \quad (9)$$

where $\phi_0 = \frac{\pi c}{e}$ is the magnetic flux quantum and $\zeta(x)$ is *zeta*-Riemann function. It is important that Eq. (8) can be analytically solved [1] and expression for the GL upper critical field slope can be analytically written:

$$H_{c2}^\parallel = \tau\left(\frac{\phi_0}{2\pi\xi_\parallel\xi_\perp}\right) = \tau\left[\frac{8\pi^2 c T_c^2}{7\zeta(3) e v_F t_\perp c^*}\right]. \quad (10)$$

To consider the Eq. (7) at $T = 0$, it is convenient to introduce the following new variables,

$$\tilde{z} = \frac{\sqrt{2t_\perp\omega_c}}{v_F}(x - x_1), \quad \tilde{x} = \frac{\sqrt{2t_\perp\omega_c}}{v_F}x, \quad (11)$$

and rewrite Eq. (7) at $T = 0$, using new variables, as

$$\Delta(\tilde{x}) = g\Big\langle \int_{\frac{\sqrt{2t_\perp\omega_c}}{v_F}d}^\infty \frac{d\tilde{z}}{\tilde{z}}\, J_0[\tilde{z}(2\tilde{x} + \tilde{z}\sin\alpha)] \times$$
$$\times \Delta(\tilde{x} + \tilde{z}\sin\alpha)\Big\rangle_\alpha. \quad (12)$$

Numerical solution of Eq. (12) (see Fig. 1) gives the following result for the parallel upper magnetic critical field in terms of the GL slope (10):

$$H_{c2}^\parallel(0) \approx 0.75\left[\frac{8\pi^2 c T_c^2}{7\zeta(3) e v_F t_\perp c^*}\right] = 0.75\ |dH_{c2}^\parallel/dT|_{T_c} T_c. \quad (13)$$

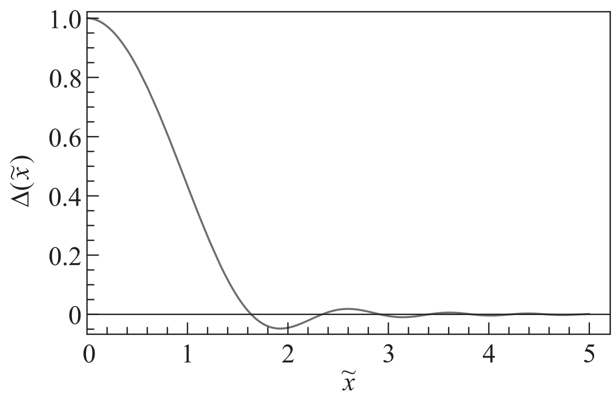

Fig. 1. (Color online) Solution of Eq. (7) for the Q2D conductor (3) in the parallel magnetic field (4) is shown. We pay attention to the fact that the solution is not of the Gaussian form, moreover it changes its sign several times with changing variable $\tilde{x}$

For the perpendicular upper critical magnetic field the corresponding results were obtained earlier [7] [see Eq. (2)]. Therefore, we predict in the Letter an increase of the Q2D anisotropy, $\gamma(T) = \left[\frac{H_{c2}^\parallel(T)}{H_{c2}^\perp(T)}\right]$, with decreasing temperature:

$$\lim_{T\to 0}\gamma(T) = \lim_{T\to 0}\left[\frac{H_{c2}^\parallel(T)}{H_{c2}^\perp(T)}\right] = 1.27 \lim_{T\to T_c}\gamma(T). \quad (14)$$

We note that, in the Letter, we have not taken into account quantum effects of electron motion along open orbits [10] and have considered case, where $t_c > T_c$, which is opposite to the Lawrence–Doniach model [11].

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364019150025

1. A. A. Abrikosov, *Fundamentals of Theory of Metals*, Elsevier Science, Amsterdam (1988).
2. L. P. Gor'kov, Sov. Phys. JETP **37**(10), 42 (1960).
3. N. R. Werthamer, E. Helfand, and P. C. Hohenberg, Phys. Rev. **147**, 295 (1966).
4. L. P. Gor'kov and T. K. Melik-Barkhudarov, ZhETF **45**, 1493 (1964) [Sov. Phys. JETP **18**, 1031 (1964)].
5. V. G. Kogan and S. L. Bud'ko, Physica C **385**, 131 (2003).
6. V. G. Kogan and R. Prozorov, Rep. Prog. Phys. **75**, 114502 (2012).
7. T. Kita, Phys. Rev. B **68**, 184503 (2003).
8. A. G. Lebed and K. Yamaji, Phys. Rev. Lett. **80**, 2697 (1998).
9. A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Mechanics*, Dover, N.Y. (1963).
10. A. G. Lebed, JETP Lett. **44**, 114 (1986) [Pis'ma ZhTF **44**, 89 (1986)].
11. L. N. Bulaevskii and A. A. Guseinov, Pis'ma ZhETF **19**, 742 (1974) [JETP Lett. **19**, 382 (1974)].