Destructive quantum interference and exceptional points in high-frequency response of two-state system

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Periodic driving transforms the stationary energy spectrum into the Floquet modes spectrum (quasienergies). This can be associated with the so-called synthetic dimension introduced by the Floquet modes [1, 2]. Perturbation frequency in this case becomes an additional degree of freedom, which opens new ways of manipulating the quantum system spectrum. In this context, periodic driving can introduce phenomena, which are typical for higher dimensional systems, in lower dimensional samples. In a finite system periodic driving can effectively change its geometry (connectivity of tunneling paths). In present letter we study interference features in the high-frequency conductance of a two-state model system. We show that the synthetic frequency dimension provides the possibility for effective degeneracy of eigenstates in a simply connected linear quantum conductor, which is impossible in statics. This is accompanied by the destructive quantum interference (DQI) and resonance coalescence, described by an exceptional point (EP), and can be observed by a dip in the real part of the conductance at resonant frequency. Our study is based on the Keldysh formalism for non-equilibrium Green functions [3] (NEGF) in tight-binding basis [4], which basics in application to the dynamical response of quantum conductors have been thoroughly developed in the 90s [5, 6].

Consider an arbitrary two-state system with the following tight-binding bare Hamiltonian:

$$\hat{H}_0 = \varepsilon_1 a_1^\dagger a_1 + \varepsilon_2 a_2^\dagger a_2 + \left(\tau a_2^\dagger a_1 + H.c. \right), \quad (1)$$

where ε_i is the on-site energy and τ is the intersite hopping. Left (right) lead is coupled to the i-th site by matrix element $\gamma_i^{L(R)}$, which is real and independent of energy within the wide-band approximation (WBA). Suppose there is a weak external AC bias applied to the

leads. Following general formalism [5, 6] one can calculate the net conductance as

$$G(\omega) = \frac{e^2}{h} \int dE \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega} \times T(E, \omega), \quad (2)$$

where $f(E)$ is the Fermi–Dirac distribution function, which is the same in the left and right lead as there is no DC bias. Factor $T(E, \omega)$ in the integrand is an energy and frequency resolved generalized transmission coefficient: $T(E, \omega) = T_{\text{Re}}(E, \omega) + iT_{\text{Im}}(E, \omega)$.

Depending on the particular values of $\gamma_i^{L,R}$ we can distinguish topologically different symmetric configurations, for instance: the linear configuration (inset in Fig. 1b) and the side-defect (SD) configuration (inset in Fig. 1d). Linear configuration corresponds to $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$, $\gamma_1^L = \gamma_2^R = \gamma$ and $\gamma_2^L = \gamma_1^R = 0$. The conductance in this case has the form (2) with

$$T_{\text{Re}}(E, \omega) = \frac{1}{2} \left[T_0^-(E, \omega) + T_0^+(E, \omega) \right]. \quad (3)$$

Here terms $T_0^\pm$ are of the form of some stationary transmission coefficients [7]:

$$T_0^\pm(E, \omega) = \frac{P_\pm^2(E, \omega)}{P_\pm^2(E, \omega) + Q_\pm^2(E, \omega)}, \quad (4)$$

with corresponding functions $P_\pm(E, \omega)$ and $Q_\pm(E, \omega)$:

$$P_\pm(E, \omega) = \Gamma(\hbar\omega \pm 2\tau),$$
$$Q_\pm(E, \omega) = \left[E - \varepsilon_0 + \frac{1}{2}\hbar\omega \right]^2 + \Gamma^2 - \frac{1}{4}(\hbar\omega \pm 2\tau)^2, \quad (5)$$

where $\Gamma = \pi\rho\gamma^2$.

Equation (4) provides a clear analysis of interference picture: zeros of functions $P_\pm$ correspond to zero-valued antiresonances (DQI) and zeros of functions $Q_\pm$ – to perfect resonances [7]. As can be seen from Eq. (5),
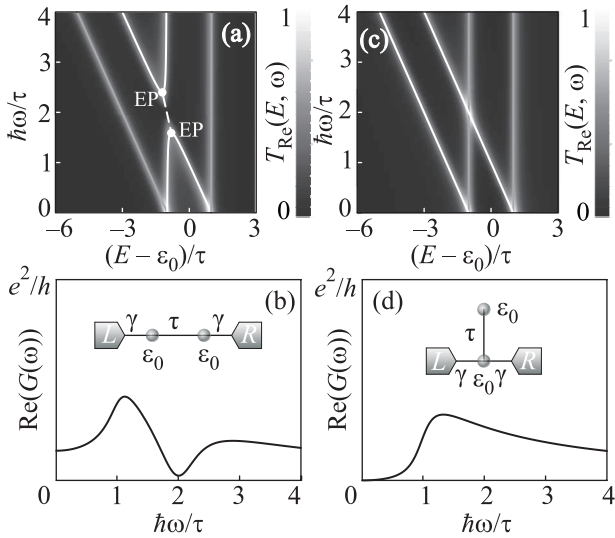
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Fig. 1. (Color online) Energy and frequency resolved real part of the generalized transmission of the two-state system in the linear configuration (a) and SD configuration (c). Real part of the conductance vs. frequency for zero constant bias, zero temperature and $E_F = \varepsilon_0$ of the linear (b) and SD configuration (d), which are shown in the insets

functions $Q_{\pm}$ can be written as characteristic determinants of some $\mathcal{PT}$-symmetric auxiliary Hamiltonians $\hat{H}^{\pm}_{\text{aux}}$: $Q_{\pm} = \det(E\hat{I} - \hat{H}^{\pm}_{\text{aux}})$, with

$$\hat{H}^{\pm}_{\text{aux}} = \begin{pmatrix} \varepsilon_0 \mp \tau - \hbar\omega & i\Gamma \\ i\Gamma & \varepsilon_0 \pm \tau \end{pmatrix}. \qquad (6)$$

For finite frequency conditions for the EPs are $\Gamma = |\tau + \hbar\omega/2|$ and $\Gamma = |\tau - \hbar\omega/2|$, i.e., AC field effectively modifies the tunneling matrix element τ, which defines the energy split between the eigenstates. For $\hbar\omega = \pm 2\tau$ this split can vanish, which resembles the case of the stationary transmission through the quantum system with degenerate states [8]. From Eq. (3) one can also see that there are antiresonances for $\hbar\omega = \pm 2\tau$ (zeroes of $P_{\mp}$), which is the manifestation of a photon-assisted DQI phenomena.

Figure 1a depicts the real part of the generalized transmission coefficient (3) calculated for $\Gamma = 0.2\tau$. According to Eq. (5), there is coalescence of resonances of $T_{\text{Re}}(E,\omega)$ for $\hbar\omega = 2\tau \pm 2\Gamma$ at energy $E = \varepsilon_0 - \tau \mp \Gamma$, which correspond to the EP of $\hat{H}^{-}_{\text{aux}}$. This causes a dip in the real part of the conductance at $\hbar\omega = 2\tau$ (Fig. 1b).

SD configuration is defined by $\gamma_1^L = \gamma_1^R = 0$, $\gamma_2^L = \gamma_2^R = \gamma$, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$. In this case we have

$$T_{\text{Re}}(E,\omega) = \frac{1}{2}[T_0(E) + T_0(E + \hbar\omega)]. \qquad (7)$$

Here $T_0(E)$ is a stationary transmission coefficient of the form (4) and functions P and Q equal to

$$P(E) = 2\Gamma(E - \varepsilon_0), \quad Q(E) = (E - \varepsilon_0)^2 - \tau^2. \quad (8)$$

$T_{\text{Re}}(E,\omega)$ of the SD configuration is illustrated by Fig. 1c. The real part of the conductance (Fig. 1d) demonstrates only peak, corresponding to the photon-assisted (absorption/emission) resonant tunneling, i.e., there is no AC field induced additional interference effects.

Equations (3) and (7) for the real parts of the generalized transmission coefficient consist of two terms, which are of the form (4) and can be regarded as some stationary transmission coefficients. It turns out that they correspond to the stationary scattering problem for the initial symmetric configuration with either symmetric or antisymmetric eigenstates shifted by $-\hbar\omega$. In the case of SD configuration both eigenstates are of the same parity – symmetric and hence they are shifted (or not) all together, what preserves the interference picture of the stationary scattering. In the linear configuration eigenstates are of opposite parity and they are shifted relative to each other and the interference picture is being modified. In this case the effective degeneracy at $\hbar\omega = \pm 2\tau$ is exactly the dynamical counterpart of the situation considered in [8]. One can treat this phenomenon by introduction of a synthetic frequency dimension that allows linear configuration to behave as multiply-connected and possess DQI.

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