Pis'ma v ZhETF, vol. 111, iss. 7, pp. 441 – 442

On dimension of tetrads in effective gravity

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Submitted 29 February 2020
Resubmitted 10 March 2020
Accepted 11 March 2020

DOI: 10.31857/S0370274X20070024

There are several scenarios of emergent gravity. Gravity may emerge in the vicinity of the topologically stable Weyl point [1–5]; the analog of curved spacetime emerges in hydrodynamics with the so-called acoustic metric for the propagating sound waves [6]; etc. Here we consider two very different scenarios, which however have unusual common property: the tetrad fields in these theories have dimension of inverse length. As a result all the physical quantities which obey diffeomorphism invariance are dimensionless. This was first noticed by Diakonov [7] and Vladimirov and Diakonov (VD) [8, 9] in the scenario, where tetrad fields emerge as bilinear combinations of the fermionic fields. Tetrads with dimension of inverse length emerge also in the model of the superplastic vacuum [10, 11].

In the theory by VD [7–9] the tetrads are composite fields, which emerge as the bilinear combinations of the fermionic fields:

$$e^A_\mu = i \left\langle \psi^\dagger \gamma^A \nabla_\mu \psi + \nabla_\mu \psi^\dagger \gamma^A \psi \right\rangle . \tag{1}$$

This construction is similar to what happens in the spin-triplet p-wave superfluids in the ^{3}He-B phase [12]. In the VD scenario two separate Lorentz groups of coordinate and spin rotations are spontaneously broken to the combined Lorentz symmetry group, $L_L \times L_S \to L$. In the same manner in ^{3}He-B the symmetries under three-dimensional rotations in orbital and spin spaces are broken to the symmetry group of combined rotations, $SO(3)_L \times SO(3)_S \to SO(3)_J$.

Formation of tetrads breaks both the symmetries under discrete coordinate transformations $P_L = (\mathbf{r} \to -\mathbf{r})$ and $T_L = (t \to -t)$, and the discrete symmetries in spin space, P_S and T_S. The symmetry breaking scheme $P_L \times P_S \to P$ and $T_L \times T_S \to T$ leaves the combined parity P and the combined time reversal symmetry T.

The VD symmetry breaking mechanism can be important for the consideration of the Big Bang scenario, in which the gravitational tetrads change sign across the singularity, $e^A_\mu(\tau, \mathbf{x}) = -e^A_\mu(-\tau, \mathbf{x})$ [13, 14]. The singularity can be avoided by formation of the bubble with a vanishing determinant of the metric [15, 16], which would correspond of the vacuum state with unbroken symmetry, i.e., with zero tetrad field, $e^A_\mu = 0$. On the other hand, the Big Bang can be considered as a symmetry breaking phase transition $L_L \times L_S \to L$, at which the symmetry between the spacetime with $e > 0$ and anti-spacetime with $e < 0$ is spontaneously broken, where e is the tetrad determinant. Correspondingly, in superfluid ^{3}He the formation of the p-wave order parameter spontaneously breaks the symmetry under coordinate transformation $\mathbf{r} \to -\mathbf{r}$. The VD scenario has also the connection to the chiral ^{3}He-A phase: in both systems the topologically protected Weyl fermions emerge, which move in the effective tetrad field [5].

According to Eq. (1), the frame field e^A_μ transforms as a derivative and thus has the dimension of inverse length, $[e^A_\mu] = 1/[l]$ (it is assumed that ψ is scalar under diffeomorphisms) [7, 8]. For Weyl or massless Dirac fermions one has the conventional action:

$$S = \int d^4x |e| e^{A\mu} \left(\psi^\dagger \gamma^A \nabla_\mu \psi + \text{H.c.} \right) . \tag{2}$$

The action (2) expressed in terms of the VD tetrads is dimensionless, since $[e] = [l]^{-4}$, $[e^{A\mu}] = [l]$ and $[\psi] = 1$.

The elasticity tetrads describe elasticity theory [10, 11, 17, 18]. In conventional crystals they are gradients of the three $U(1)$ phase fields X^A, $A = 1, 2, 3$,

$$e_\mu{}^A(x) = \partial_\mu X^A(x). \tag{3}$$

The surfaces of constant phases, $X^A(x) = 2\pi n^A$, describe the system of the deformed crystallographic planes. Being the derivatives, elasticity tetrads have also canonical dimensions of inverse length. This allows us to extend the application of the topological anomalies. The Chern–Simons term describing the $3+1$ quantum Hall effect becomes dimensionless. As a result, the prefactor of term is given by the integer momentum-space topological invariants in the same manner as in the case of 2+1 dimension.

The elasticity tetrads can be used as the gravitational tetrads for the construction of gravity in the model of the 3+1 vacuum as a plastic (malleable) fermionic crystalline medium with $A = 0, 1, 2, 3$ [19, 20].

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In plastic vacuum all physical quantities become dimensionless [11]. Such vacuum can be arbitrarily deformed, and thus the equilibrium microscopic length scale (such as Planck scale) is absent. All distances are measured in terms of the integer positions of nodes of plastic crystal, and the Newton constant, the scalar curvature R, the cosmological constant Λ, and particle masses M become dimensionless [11].

The same is for VD gravity, where "all world scalars are dimensionless, be it the scalar curvature R, the interval ds, the fermion field ψ, or any diffeomorphism-invariant action term" [8]. Example is the mass term:

$$S = \int d^4x|e|M\psi^\dagger\psi, \qquad (4)$$

$[e] = [l]^{-4}$, $[\psi] = 1$ and $[M] = 1$. For bosonic scalar field

$$S = \int d^4x\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\Phi\nabla_\nu\Phi + M^2\Phi^2\right), \qquad (5)$$

one has $[g^{\mu\nu}] = [l]^2$, $[\sqrt{-g}] = [l]^{-4}$, $[\Phi] = 1$ and $[M] = 1$.

In both scenarios of emergent gravity, the dimensionless physics is supported by the invariance under diffeomorphisms. In the VD theory this invariance is assumed as fundamental. In the superplastic vacuum, it is invariance under deformations of the 4D crystal. All this suggests that the dimensionless physics can be the natural consequence of the diffeomorphism invariance, and thus can be the property of the gravity, which we have in our quantum vacuum.

Note the difference with the conventional expression of the physical parameters in terms of the Planck units, where the Newton constant $G = 1$, and all the physical quantities also become dimensionless. In this approach the masses of particles are expressed in terms of the Planck energy, which is assumed to be the fundamental constant. However, in principle the Planck energy or the Newton constant may depend on the trans-Planckian physics, and thus can (and should) be space and coordinate dependent. This occurs in the modified gravity theories, such as the scalar-tensor and $f(R)$ theories (see, e.g., [21]), and in the so-called q-theory [22]. While in the VD approach the "fundamental constants" do not exist, and only dimensionless ratios and the topological quantum numbers make sense. Then, instead of the fundamental constants, the most stable physical quantities should be used.

The dimensionless physics emerging in the frame of the VD dimensionful tetrads leads to new topological terms in action. Some of the dimensionless parameters appear to be the integer valued quantum numbers, which describe topology of quantum vacuum. Example is the 3+1 dimensional quantum Hall effect in topological insulators [11]. When the Chern–Simons action is written in terms of the elasticity tetrads with $[e^A_\mu] = 1/[l]$, its prefactor becomes dimensionless and universal, being expressed in terms of integer-valued momentum-space invariant. The relativistic example is the chiral anomaly in terms of torsion fields [23, 24]. For the torsion and curvature in terms of the conventional tetrads, the gravitational Nieh–Yan anomaly equation for the non-conservation of the axial current

$$\partial_\mu j^\mu_5 = \lambda^2\left(\mathcal{T}^A \wedge \mathcal{T}_A - e^A \wedge e^B \wedge R_{AB}\right), \qquad (6)$$

contains the nonuniversal prefactor – the ultraviolet cut-off parameter λ with dimension $[\lambda] = 1/[l]$, which may depend on the spacetime coordinates, explicitly violating the topology. In terms of VD tetrads, the prefactor λ becomes dimensionless, $[\lambda] = 1$, which properly reflects the topology of the quantum vacuum.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364020070024

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