

Spin vortex lattice in the Landau vortex-free state of rotating superfluids

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In the rotating vessel the lattice of mass vortices represents the ground state or the thermal equilibrium state of the rotating superfluid. For spin vortices the situation is different. Orbital rotations do not act on the spin vortices, and the other external or internal fields are needed for the formation of the lattice, such as Dzyaloshinskii–Moriya interaction [1, 2], which leads to the formation of skyrmion lattice. Here we show that the lattice of spin vortices can be created in rotating vessel, if the formation of the mass vortices is suppressed. The Landau vortex-free state in the rotating vessel (the analog of Meissner state in superconductors) acts on spin vortices as rotation acts on mass vortices.

We start with the Bose–Einstein condensation of magnons (magnon BEC), which is realized in superfluid ^{3}He-B as the homogeneously precessing domain (HPD) [3–5]. Magnon BEC is characterized by the density of magnons $n_{\rm M} = S - S_z$, where $S = \chi H$ is spin density in magnetic field; S_z is the projection of the precessing spin on magnetic field, see review [6]. The magnon BEC has the superfluid velocity

$$\mathbf{v}_{\rm M} = \frac{\hbar}{m_{\rm M}} \boldsymbol{\nabla}\alpha, \qquad (1)$$

where α is the angle of the precession, which plays the role of the phase of magnon BEC; and $m_{\rm M}$ is magnon mass. In spin dynamics, the magnon density $n_{\rm M}$ and the phase α are canonically conjugate variables, and thus

$$\mathbf{P} = n_{\rm M} \boldsymbol{\nabla}\alpha = m_{\rm M} n_{\rm M} \mathbf{v}_{\rm M}, \qquad (2)$$

represents the momentum density of the magnon field.

In the moving superfluid the magnon BEC acquires the Doppler shift energy term:

$$F_{\rm mix} = \mathbf{P} \cdot (\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}) = m_{\rm M} n_{\rm M} \mathbf{v}_{\rm M} \cdot (\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}). \qquad (3)$$

Here $\mathbf{v}_{\rm s}$ and $\mathbf{v}_{\rm n}$ are correspondingly superfluid and normal velocities of the B-phase. This term, which mixes superfluid velocity of the background mass superfluid and superfluid velocity of magnon BEC, is another realization of the Andreev–Bashkin effect in superfluid mixtures, when the superfluid current of one component depends on the superfluid velocity of another component [7]. The counterflow $\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}$ together with spin density plays the similar role as Dzyaloshinskii–Moriya interaction in magnets, which violates the space inversion symmetry and leads to formation of skyrmion lattices, see, e.g., [8]. The mixed term modifies the kinetic energy:

$$F_{\rm grad} = \frac{1}{2} m_{\rm M} n_{\rm M} \mathbf{v}_{\rm M}^2 + m_{\rm M} n_{\rm M} \mathbf{v}_{\rm M} \cdot (\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}). \qquad (4)$$

We consider the Landau state in the container rotating with angular velocity $\boldsymbol{\Omega}$, when the normal component of the liquid has the solid body rotation with velocity $\mathbf{v}_{\rm n} = \boldsymbol{\Omega} \times \mathbf{r}$, while the superfluid component of the B-phase is vortex-free, $\mathbf{v}_{\rm s} = 0$, and thus

$$F_{\rm grad} = \frac{1}{2} m_{\rm M} n_{\rm M} \left(\mathbf{v}_{\rm M} - \boldsymbol{\Omega} \times \mathbf{r}\right)^2. \qquad (5)$$

This means that the Landau state in the rotating vessel acts on spin superfluid (magnon BEC) in the same way as rotation acts on mass superfluid, i.e., it should lead to formation of spin vortices, in which the phase α has 2π winding (single spin vortex has been constructed and identified [9]). So, if the creation of mass vortices is suppressed, but the creation of spin vortices is allowed, one obtains the state with the lattice of spin vortices.

The number of these spin vortices in the Landau state in rotating vessel is determined by the circulation quantum of spin vortex $\kappa_{\rm M} = 2\pi\hbar/m_{\rm M}$ and by angular velocity. That is why the number of spin vortices in the lattice in the Landau state can be expressed in terms of the equilibrium number of quantized mass vortices in the fully equilibrium rotating state:

$$\frac{N_{\rm spin}}{N_{\rm mass}} = \frac{\kappa_3}{\kappa_{\rm M}} = \frac{m_{\rm M}}{2m_3}, \qquad (6)$$

where $\kappa_3 = 2\pi\hbar/2m_3$ is the quantum of circulation in superfluid ^{3}He-B; m_3 is the mass of ^{3}He atom; magnon

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mass is $m_{\rm M} = \omega_L/2c_{\rm s}^2$, where ω_L is Larmor frequency, and $c_{\rm s}$ is the speed of spin waves in ^{3}He-B [6]. So we have two rotating states: the fully equilibrium rotating state where the mass vortices form the vortex lattice, while the spin vortices are absent; and the metastable Landau state in rotating vessel, where mass vortices are absent, while spin vortices form the lattice.

Similar effect of the formation of the vortex lattice takes place for the conventional spin vortices in the polar phase of superfluid ^{3}He. This phase exists in the nanoscale confinement (in the so called nafen) [10–21]. The Landau states have been observed in the polar phase in spite of zero value of the Landau critical velocity in this superfluid with Dirac nodal line [22]. According to Brauner and Moroz [23], in the presence of both the counterflow $\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}$ and magnetic field $\mathbf{H}$, the spin texture is formed, which originates form the similar mixed term in Eq. (3):

$$F_{\rm mix} = S\boldsymbol{\nabla}\alpha \cdot (\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n}), \tag{7}$$

where α is the angle of the unit $\hat{\mathbf{d}}$-vector, which describes the spin part of the order parameter, and $S = \chi H$ is spin density in magnetic field. In the Landau state of the polar phase in the rotating cryostat, with $\mathbf{v}_{\rm n} = \boldsymbol{\Omega} \times \mathbf{r}$ and $\mathbf{v}_{\rm s} = 0$, the gradient energy for spin textures is:

$$F_{\rm grad} = \frac{1}{2}\rho_{\rm spin}\left(\boldsymbol{\nabla}\alpha - \frac{S}{\rho_{\rm spin}}\boldsymbol{\Omega} \times \mathbf{r}\right)^2, \tag{8}$$

which should lead to the lattice of spin vortices. The number of equilibrium spin vortices in the Landau state in the vessel of radius R is:

$$N_{\rm spin} = \frac{\chi H}{\rho_{\rm spin}}\Omega R^2. \tag{9}$$

So, in the spin-triplet superfluids, such as superfluid ^{3}He [24, 25], the Landau state of the superfluid in the rotating container can be the source of the formation of the vortex lattice of spin vortices. For the experimental realization of spin-vortex lattice the sufficently large magnetic field is required. Similar phenomenon may occur in rotating neutron stars (review on superfluidity and superconductivity in neutron stars see in [26]).

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