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Numerical simulation of collinear capillary-wave turbulence

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We report on direct numerical simulation of quasi-one-dimensional bidirectional capillary-wave turbulence. Although nontrivial three-wave and four-wave resonant interactions are absent in this peculiar geometry, we show that an energy transfer between scales still occurs concentrated around the linear dispersion relation that is broadened by nonlinearity. The wave spectrum displays a clear wave number power-law scaling that is found to be in good agreement with the dimensionally prediction for capillary-wave turbulence involving four-wave interactions. The carried out high-order correlation analysis (bicoherence and tricoherence) confirms quantitatively the dominant role of four-wave quasi-resonant interactions. The Kolmogorov–Zakharov's (KZ) spectrum constant is also estimated numerically. We interpret our results as the first numerical observation of anisotropic capillary-wave turbulence in which four-wave interactions play a dominant role.

Let us now rewrite the wave-elevation spectrum prediction of weak turbulence obtained by dimensional analysis following [1]. Introduce the wave elevation power spectrum as $S(k) = |\eta_k|^2$, where η_k is the spatial Fourier transform of the wave elevation $\eta(x)$ at the location x. Assuming that the leading order process is three-wave interaction, we obtain the prediction of the anisotropic (quasi-1D) capillary-wave turbulence spectrum

$$S(k) = C_{\rm 1D}^{\rm 3w} P^{1/2} \left(\frac{\sigma}{\rho}\right)^{-3/4} k^{-15/4}, \quad (1)$$

$$S(\omega) = \frac{2}{3} C_{\rm 1D}^{\rm 3w} P^{1/2} \left(\frac{\sigma}{\rho}\right)^{1/6} \omega^{-17/6}, \quad (2)$$

where $C_{\rm 1D}^{\rm 3w}$ is the KZ constant, P is the energy dissipation rate, σ is the surface tension, ρ is the mass density of the fluid, and ω is the angular frequency. Equation (2) being obtained from (1) with $S(k)dk = S(\omega)d\omega$. The expressions (1) and (2) are similar to the well known Zakharov–Filonenko spectrum of isotropic capillary wave turbulence [2]. Equations (1)–(2) are obtained under the assumption of the dominant influence of three-wave interactions.

At the next order (four-wave interactions), the anisotropic capillary-wave turbulence spectrum reads

$$S(k) = C_{\rm 1D}^{\rm 4w} P^{1/3} \left(\frac{\sigma}{\rho}\right)^{-1/2} k^{-7/2}, \quad (3)$$

$$S(\omega) = \frac{2}{3} C_{\rm 1D}^{\rm 4w} P^{1/3} \left(\frac{\sigma}{\rho}\right)^{1/3} \omega^{-8/3}. \quad (4)$$

Note that the predictions of (3)–(4) involving four-wave interactions differ from the ones of (1)–(2) obtained for the three-wave system. The difference in the k-power-law exponent is roughly 7 %: $-15/4 = -3.75$ (three-wave system) vs. $-7/2 = -3.5$ (four-wave system). The aim of this work is to perform direct numerical simulation of anisotropic capillary-wave turbulence with high accuracy.

The numerical model used in the present work is based on the cubic nonlinear equations of the boundary motion. The numerical integration scheme for the solution of the governing equations in time is based on the fourth-order explicit Runge–Kutta method. The spatial derivatives and integral operators are calculated using the pseudo-spectral methods. The numerical simulation results indeed show that a capillary-wave turbulence regime is observed in the peculiar 1D bidirectional geometry, once a high enough level of pumping is reached.

Figure 1 shows the time-averaged spatial-power spectrum $S(k)$ of the wave height $\eta(x,t)$ in the stationary state. A clear power-law scaling is observed on more than one decade in k. The best fit is $S(k) \sim k^{-3.5\pm0.1}$ which is closer to the prediction of (3) than to the

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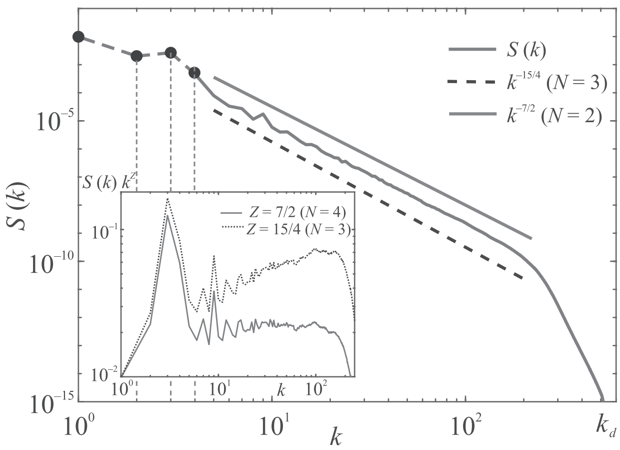

Fig. 1. (Color online) Time-averaged spatial spectrum $S(k)$ of wave elevation in the quasi-stationary state. Black bullets correspond to the harmonics pumped. Solid line corresponds to (3), dashed line to (1). Inset: compensated spectra $S(k)k^{15/4}$ and $S(k)k^{7/2}$ vs. k

one of (1). The compensated spectra are shown in the inset of Fig. 1. Indeed, the wave turbulence spectrum $S(k)$ is better approximated by $k^{-7/2}$ than by $k^{-15/4}$ within the inertial range $5 < k < 180$. Thus, our direct numerical simulations suggest that four-wave quasi-resonant interactions are involved for 1D capillary-wave turbulence. We hope that our study will trigger future investigations, notably to better understand the large-scale dynamics (larger than the forcing scale) of collinear wave turbulence such as the inverse cascade or the statistical equilibrium.

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